DOCUMENT RESUME

ED 275 882 CE 045 485

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TITLE Teaching Mathematical Trade Topics for Conceptual Understanding.
INSTITUTION TAFE National Centre for Research and Development, Payneham (Australia).
REPORT NO ISBN-0-86397-249-7
PUB DATE 86
NOTE 76p.
AVAILABLE FROM Nelson Wadsworth, P.O. Box 4725, Melbourne, Victoria, 3001 Australia.
PUB TYPE Reports - Research/Technical (143) -- Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01/PC04 Plus Postage.
DESCRIPTORS Classroom Techniques; *Concept Formation; *Concept Teaching; Electrical Occupations; Foundries; *Mathematics Skills; Roofing; Teaching Methods; *Technical Mathematics; *Trade and Industrial Education

ABSTRACT

This book is intended to assist trade and industrial education teachers in teaching mathematical trade topics in a way that will increase students' conceptual understanding of them. The first chapter provides an overview of the book's contents and suggests ways of using it. The next five chapters address the following aspects of using principles and methods to increase students' conceptual understanding of the material at hand: mathematical methods (the mathematical and conceptual approaches); the topic plan (learning in context; modeling, solving, and interpreting; identifying facts, skills, strategies, and concepts; using prior knowledge; choosing an example, and lesson planning); teaching the facts and skills; teaching the strategies; and teaching the concepts (method for teaching concepts; development of a concept; workshop experience; discussion and analysis; generalization, representation, and reinforcement of a concept; application, and recording procedures). Chapters 7 through 9 contain examples specific to the teaching of roofing, electrical, and foundry applications. Each of these chapters contains sections addressing some or all of the following: context; analysis of the task; modeling, solving, and interpreting techniques; analysis of the mathematics; lesson planning procedures; review; facts and skills; concepts; and strategies. A resource wedge that groups resource types in a hierarchy based on their degree of concreteness or abstraction is appended. (MN)

TAFE NATIONAL CENTRE FOR
RESEARCH AND DEVELOPMENT

TEACHING MATHEMATICAL TRADE TOPICS FOR CONCEPTUAL UNDERSTANDING

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and
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ADELAIDE 1986

ISBN 0 86397 249 7
TD/TNC 13.3

Published by:

TAFE National Centre for
Research and Development
296 Payneham Road
Payneham SA 5070

(Incorporated in South Australia)

Distributed by Nelson Wadsworth, PO Box 4725, Melbourne VIC 3001
for TAFE National Centre for Research and Development Ltd.

Printed by D. J. WOOLMAN, Government Printer, South Australia

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ACKNOWLEDGEMENTS

The authors wish to thank Dr John Conroy, Mr Graham Hermann, Ms Jeannette Thiering, and others too numerous to list for their assistance and for their comments on earlier drafts of this document.

While their assistance is gratefully acknowledged, full responsibility for the contents of this publication is accepted by the authors.

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CHAPTER ONE: INTRODUCTION

Teachers of trade courses frequently find that their students have difficulty with the solving of problems associated with the more abstract topics. When this occurs the teacher needs to become a teacher of mathematics, as well as being a teacher of the trade concerned.

We believe that many such difficulties arise because the students, in their previous courses, have become skilled at the various operations and processes without really understanding what they are doing. Their skills have enabled them to 'pass' examinations based on exercises with which they are familiar. However they have difficulty in applying their knowledge to unfamiliar situations such as those they meet on the job. The solution to the problem is to start the students thinking about what they are really doing when they perform some mathematical skill.

Our first objective is to illustrate how to analyse a mathematical topic by identifying its component facts, skills, strategies and concepts. A method suitable for teaching each at these elements is suggested. However the process of analysing a topic using this or any other classification scheme should not be allowed to interfere with the primary consideration, which is to provide an adequate quantity and quality of learning experiences for the students to achieve adequate understanding.

Before a trade problem can be solved mathematically it must be expressed using mathematical language. We refer to this translation process as modelling. After a solution has been found it has to be translated back from the mathematical form into everyday terms. This process is referred to as interpretation.

Students frequently have difficulties because elementary mathematics courses emphasise the process of finding a solution. Students are given precisely enough information to find a solution. The <u>problem</u> cannot be solved if insufficient information is provided. Teachers of trade mathematics must consider the modelling of their problems in mathematical terms, as well as the interpretation of the solution, because these aspects may be new to their students.

Our second objective is to show how to incorporate this view of a problem solving process into the teaching methods, by identifying the modelling, solving and interpretation stages in the analysis of a problem.

HOW TO USE THIS BOOK

Because mathematical topics are included in a wide variety of courses it is unlikely that teachers will find a ready made solution provided for the exact problems or topics they need to teach.

The material in chapters 2 to 6 is presented in a way which will enable teachers to identify and apply the correct principles to their future teaching of mathematical topics.

Chapter two contains definitions of the terms used and a summary of the teaching methods suggested.

Chapter three outlines the planning and development of a unit of work.

Chapters four to six expand on the summary of teaching methods presented in chapter one.

Chapters seven to nine contain examples specific to the teaching of particular topics. Teachers should read one of these if they consider that it is sufficiently related to their area of interest.

After reading chapters two and three teachers should be able to plan their next unit of work with regard to:

- the trade context;
- the modelling solving and interpretation aspects;
- the facts skills strategies and concepts involved;
- the prior knowledge of the students;
- the use of suitable examples.

In the absence of a ready made formula for teaching everything to all students we believe that teachers with an interest in improving the quality of the learning experiences they provide will read the ideas presented here and use as many of them as are relevant.

The major purpose of teaching mathematics in trade courses is to enable TAFE students to solve problems encountered on the job. In each work situation, there are many different problems to be solved. Each of these problems may be solved in more than one way. Trade persons traditionally have preferred methods consisting of small steps which are easily performed in a sequence. Mathematical methods provide an alternative for some of these steps, when more suitable methods are not available.

If the appropriate approach to the problem is not stated in a syllabus document, a major decision to be made by the TAFE lecturer concerns which of the available methods to use. For example, problems of roof construction are solved using the steel square, Hancock tables, calculation, or by laying out on the ground at the building site. Each of these approaches uses different mathematical facts, skills, strategies and concepts. The approach selected by TAFE lecturers should be one using mathematical abilities which <u>their</u> particular group of students has already developed, either in school mathematics and science classes, in other TAFE courses or in earlier classes.

Mathematics taught in a classroom may become divorced from the job context if care is not taken by the teacher to specifically relate the two. Without seeing the relationship students have difficulties in the abstract skills learned in the classroom to the practical situation. Additionally, the relevance of the material being learned becomes less obvious to the student. Lack of motivation becomes a problem. Hence, mathematical skills should be taught in the context of a particular trade which is of interest to the students. These mathematical skills should be related to the specific job on hand by the use of trade terminology rather than mathematical terminology. The units of measurement which should be used are those which are most widely used on the job.

REASONS FOR USING A MATHEMATICAL APPROACH

Many tasks which are performed by trade persons are completed by drawing on the expertise and background knowledge of the experienced and skilled operator. Students attempting the same tasks may need to resort to arithmetic or algebraic calculation to compensate for their lack of practical experience.

Mathematical methods of solving problems are frequently more manageable and more efficient for students than traditional trade methods in situations where a small calculation replaces a great deal of measurement. Calculation leads to the use of more efficient techniques, and methods which may be more easily understood and learned by the students.

Trade persons relying on rules of thumb, supported by years of trade experience, may experience difficulties when they are endeavouring to teach more formal and analytical methods based on rules and formulae. When there is technological change, existing tables, rules and methods cease to be applicable, and the more mathematical approach can be used to generate new rules and procedures which are more appropriate to the new technology.

Some areas of mathematics such as calculation, measurement, estimation and approximation, geometry of plan figures and spatial knowledge are basic to many trades. These contain many of the basic skill areas which students need to master, in order to be able to cope with the theoretical and practical aspects of their trade.

THE CONCEPTUAL APPROACH

In many situations it is desirable for students to develop an understanding of what they are doing, as well as to develop the skills for doing the job. In these situations the conceptual approach is justified. Most mathematical ideas are not difficult. If the focus of the teaching process is to develop concepts, students are better able, through their understanding of the underlying principles, to solve unfamiliar or unusual problems such as those met in new job situations.

If the mathematical elements of a trade problem are taught only by showing the students a formula, then using it to perform routine calculations, there is a greater chance that they will have difficulty in applying the skills in the job situation. For example, some students will not be able to decide when the particular formula is applicable. Even when the answer is obtained, some will not be able to relate the numerical answer to the job. That is, they will not be able to interpret the answer in terms of the decisions which they have to make in doing their job.

The following method for teaching basic concepts has been developed in an effort to overcome these problems. It relies on two beliefs. The first of these is a belief that teaching methods which are based on conceptual understanding are more effective than methods involving rote memorisation. The second belief is that difficulties associated with the transfer of skills to new problems and different trade situations are minimised if skills are taught in the context of particular trade applications.

The comment is sometimes made that there is not enough time in trade courses for the development of mathematical concepts. We contend that if the topic is worth teaching at all, it is worth teaching properly. That is to say, time taken to teach topics without understanding is time wasted in the longterm.

SOME DEFINITIONS

Recent research indicates that there are four identifiable elements to be considered in the teaching of mathematics. The four elements are facts, skills, strategies, and concepts. Three different methodologies are recommended, in the following section, for the teaching of these elements.

Facts

Facts are items of information which are essentially unconnected, and which do not follow logically from any other knowledge or understanding.

Skills

Skills are procedures which it is possible to carry out by the use of a routine.

Strategies

Strategies are knowledge or procedures that guide the students' choice of which skills to use or what knowledge to draw upon at each stage in the course of solving a problem.

Concepts

Concepts are connected abstract bodies of knowledge which include all the main features of a topic or idea.

Mathematical concepts represent the pattern or structure of the mathematics. They may be simple or complex. For example, the concepts of addition, matrix inversion and integration are all concepts of operations yet they differ in their nature and complexity.

Consideration of a few concepts makes it clear that there is a verbal component for each concept. We use words and symbols to describe concepts. Some of the words are everyday words but there are many technical words which are unique for each concept.

The essential difference between facts, skills, strategies and concepts is illustrated by the examples in Figure 1. One example relevant to each of the trade areas discussed in chapters 7, 8 and 9 is included.

EXAMPLES

Roofing

Concepts

Angles; lines; triangles.
Ratios (of 2 sides)

$$\text{Sine } x^{\circ} = \frac{\text{length of opposite side}}{\text{length of hypotenuse}}$$

Facts

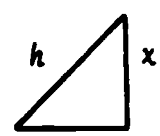
i) Sin $30^{\circ} = 0.5000$ (obtainable from calculator or tables)

ii) hypotenuse = longest side

iii) $\text{Sin } x^{\circ} = \dfrac{\text{opposite}}{\text{hypotenuse}}$

Skills

i) transposing formula
e.g. $0.5000 = \dfrac{x}{h} \quad \therefore \quad h = \dfrac{x}{0.5000}$



ii) substitution
if $x = 3.500$ then $h = \dfrac{3.5000}{0.5000}$
$\therefore \quad h = 7.000$

iii) interpretation
the length of rafter (hypotenuse) will be 7.00 metres.

Strategies i) choosing trigonometry
<u>or</u> pythagonus' Theorem
<u>or</u> tables of roofing e.g.Mancock's Tables

ii) using triagles to represent the roof.

Electrical

Concepts i) <u>Quantities</u>
current, resistance, potential difference

ii) <u>Units</u>
ampere, ohm, volt

iii) $I \propto \frac{1}{R} \longrightarrow I = \frac{V}{R}$

i.e. Current varies immersely as resistance based on concepts for V, I, & R.

i.e. doubling resistance effectively halves the current.

Facts i) Ω is the symbol for 'ohm'

ii) Resistance is measured in 'ohms'

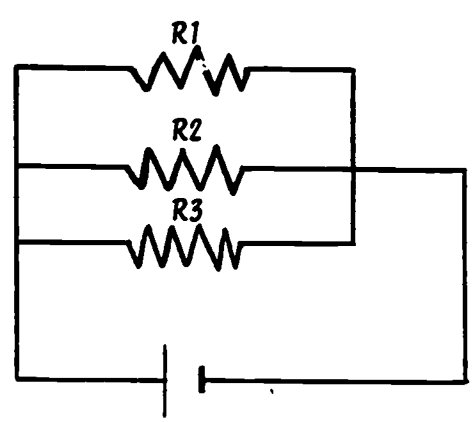
Skills i) transposing $I = \frac{V}{R} \longrightarrow V = I.R$

ii) Substitution
if $I = 3$, $V = 12$ $R = \frac{V}{I} = \frac{12}{3}$

iii) Interpretation Resistance is 4 ohms

Strategies i) Drawing a 'Circuit Diagram' to represent the situation

ii) Let R_1, R_2, R_3 be the resistances



Foundry

Concepts	i)	symbols (pronumerals e.g. x, y, z)
	ii)	operations (X, ÷, +, -)
	iii)	parts, wholes
	iv)	ratios/proportions
	v)	percentage
Facts	i)	% represents percentage
	ii)	1% is equivalent to $\frac{1}{100}$ th. part
Skills	i)	transposing a formula
	ii)	substitutions
Strategies	i)	use algebraic symbols (pronumerals) e.g. x, y, 9, d
	ii)	set up equations
	iii)	solve simultaneous equations.

Figure 1. Examples of facts, skills, strategies and concepts

Concepts, facts, skills and strategies are closely interrelated. Confusion can result when we attempt to illustrate each of these things. The ideas are important while the distinctions between them are not.

When we meet a new topic for the first time we are introduced to new concepts and old ones, an introduction to the study of weather, for example, brings discussion of old concepts of hot and cold, wet and dry, and the like. We then meet whatmay be new concepts such as summer (hot and dry in Western Australia) and winter cold and wet in Southern Australia).

Once the concepts have been developed and the new student has mastered the language involved. The ideas can be stared away as facts, e.g. It is cold and wet in Melbourne in the winter.

The objective of the TAFE lecturer should be to teach the topic in ways which will connect the new information to the students' existing conceptual structures.

TEACHING METHODOLOGIES

The same methodology is recommended for teaching facts and skills. It consists of two parts:

- establishing understanding of the fact or skill by connecting it with the students' previous experience;
- increasing the students' familiarity with the fact or skill with drill and practice to improve the fluency and confidence of their application of the newly acquired abilities.

When strategies are being taught:

- the students should practise solving problems related directly to the trade concerned;
- investigations of all the available strategies should be conducted to illustrate which lead to solutions of the problem and which do not.

Projects related to the job enable students to approach a problem confidently and with the expectation that at least one solution exists and can be found.

When concepts are being developed:

- discussion is the essential feature of the methods used because of the verbal nature of the mathematical ideas being presented;
- the students' existing concepts should be informally evaluated and any apparent weaknesses should be investigated further and if possible corrected through discussion;
- discussion of concrete practical examples and materials should lead to the development of abstractions;
- discussion of familiar situations should precede discussions of unfamiliar situations;
- the TAFE lecturer may use a site visit, hands on experience or excursion to introduce a group of students to a new trade situation;
- homework and practice exercises should be constructed with the objective of developing concepts;

- tests should be constructed to ensure that the items measure understanding and not just recall of facts which have been memorised. Students should be required to explain their answers. Care should be taken not to make the tests too difficult to read;

- conflicts of understanding should be resolved through discussion and illustration.

The appendix contains a scheme for classifying resources which might be of use to TAFE lecturers.

There are six aspects of preparation which TAFE lecturers might consider when a plan for the teaching of a mathematical topic is being developed. These are:

- the context in which the mathematics will be applied;
- the modelling, solving and interpretation which must be done;
- identifying the facts, skills, strategies and concepts which the students are to develop;
- assessing the prior knowledge which students have about the new topic;
- choosing suitable examples for use in explaining the topic;
- planning lessons.

CONTEXT

School maths is frequently taught out of context in that the practical application, the mathematics is not considered. Hence the first consideration in developing the topic plan is to determine the context in which the mathematics will be applied.

When the mathematical topics are being taught as they are needed in an integrated course, the context is immediately obvious. The mathematical skills should be taught in this context, using the units which are commonly applied on the job, and using the trade and technical terms which are commonly used in the industry. When the mathematical topics are being taught as a separate mathematics subject, independently of the trades to which they will be applied, the TAFE lecturer should choose examples relevant to these trades.

For example, in a case where cottage construction is the context in which the mathematics will be used by some of the students, a commonly used pitch and a realistic span should be chosen. Trade terms such as pitch, rise and run should be used in the calculations instead of symbols such as the letters of the Greek alphabet, which are commonly used in secondary school mathematics classes. When the standard unit used in the industry is millimetres, the problems should be specified in millimetres. If conversions must be made frequently between metric and imperial units of measurement, examples requiring this skill should be included.

Mathematical knowledge should be related to the students' experiences and to their current work needs. When mathematical knowledge is presented to students in context as a tool rather than as an end in itself, two benefits are expected. Firstly, fewer problems should be encountered by the students in transferring their mathematical knowledge to the job on hand. Secondly, students should be able to see the direct relevance of the mathematical knowledge to their trade, and be motivated to learn and apply the mathematics.

MODELLING, SOLVING AND INTERPRETING

The second consideration in developing the topic plan is to identify the modelling, solving and interpretation which the students must perform in order to arrive at a correct mathematical solution to a practical problem. In order to recognise these stages, TAFE lecturers should work through each problem in the same way that they would expect the students to proceed. The problem should be solved using only the knowledge which the students would be expected to use. For example, the students may be required to estimate the length of timber needed for each of the common rafters in a roof. This requirement determines the particular model to be used.

The <u>modelling</u> stage involves a translation from the job situation to the 'artificial' abstractions used in the mathematical process. Modelling need not involve the drawing of diagrams or the construction of scale models. These types of strategy are applicable to the particular roofing example which follows. However the mathematical model for the metallurgy problem in Chapter 9 is a pair of equations. In the construction of a building, the roof is first modelled as a simple drawing, as shown in Figure 2.

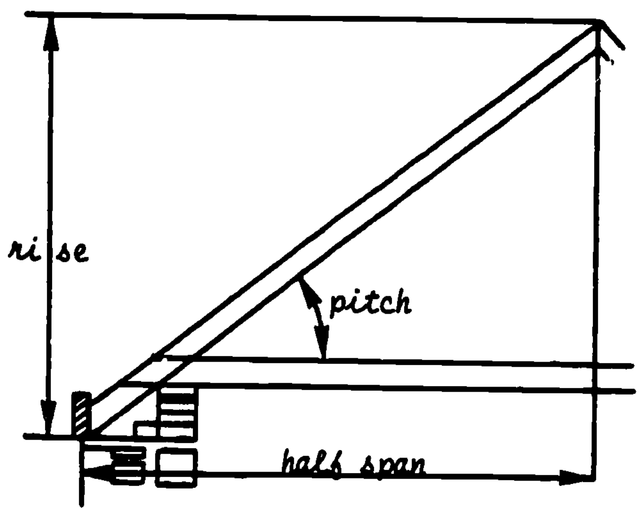


Figure 2. Roof section

Secondly, it is modelled as a triangle, as shown in Figure 3.

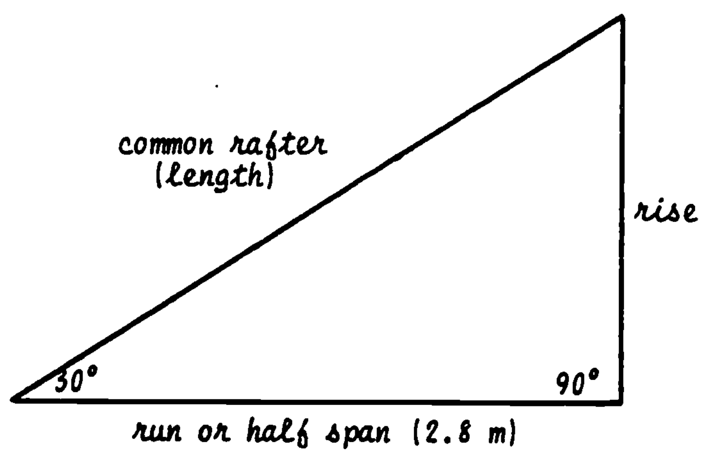


Figure 3. Triangle model

It is important for the students to take as many such steps as is necessary for them to be able to visualise and understand the relationships between the real roof and the drawings of it. If the students are able to do this well, the interpretation stage, which comes later, will cause them fewer problems. Students must recognise, for example that the hypotenuse of the triangle in Figure 3 is not parallel to the centre line of the rafter.

School mathematics is not based on solving practical problems. Students are usually given no more than the necessary amount of information to solve a problem. Hence TAFE lecturers need to show students how to extract the appropriate information from the job situation.

The <u>solving</u> stage involves the calculation of an abstract solution from the model.

This will be correct if, and only if:

- the model is correct in that it adequately represents the real situation;
- the solving process is performed correctly.

For example, to find the length of the common rafter, the solving stage might be:

$$\cos 30 = \frac{2.8}{\text{rafter length}}$$

so the $\quad \text{rafter length} = \dfrac{2.8}{\cos 30}$

that is $\quad$ rafter length = 3.233 (by calculator, or tables and division).

The <u>interpretation</u> stage involves the translation and interpretation of this abstract solution back to the form required on the job. As this stage is not emphasised in school mathematics courses, TAFE lecturers must be aware of the need to teach it. For example, the triangle used in the second step of the modelling stage has been drawn so that the length of its hypotenuse is greater than the length of the timber required for the rafter.

It can be seen from Figure 4 that while the solution above is mathematically correct the result would be wrong if it was used to mark out the cuts on the piece of timber, without reducing the rafter length to allow for half the thickness of the ridge. Figure 4 illustrates the relationship between the sketch in Figure 2 and the triangle in Figure 3. The solution must be interpreted accordingly.

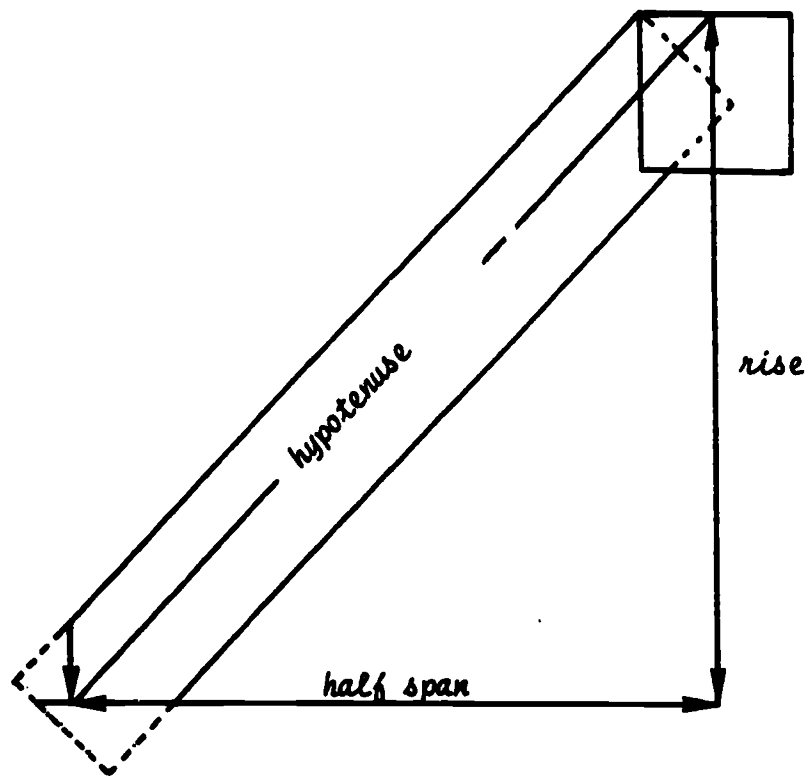


Figure 4. Position of hypotenuse

This example shows the importance of modelling. If the model is inadequate the answer will be wrong even if all the solving steps are done correctly. If the modelling and interpretation are recognised as distinct stages of the mathematics, problems such as this are easier to discuss and avoid.

The interpretation stage should also involve the estimation of the answer to check the correctness of the calculation. If should also be noted that when the skill of estimation of approximate answers is being taught it may be better to estimate the answer before the calculation is made. However, in this case estimation is being used as a tool to assist the students to interpret their solutions, so it is more logical to do the calculation first. TAFE lecturers should insist on estimations being made, either before or after calculating, particularly if students are using calculators.

At this point, it should be noted that there are better methods than calculation for the actual measuring and marking out of the rafters. However, it is not necessary to go through this marking out process to obtain an accurate estimate of the length of timber required.

IDENTIFYING FACTS, SKILLS, STRATEGIES AND CONCEPTS

The third consideration in developing the topic plan is to list the mathematical facts, skills, strategies and concepts which the students will need to develop. If the topic is an unfamiliar one for the TAFE lecturer, the list of the facts, skills, strategies and concepts might be developed by discussion with other TAFE lecturers, by consultation with a specialist mathematics teacher, by consultation with a manufacturer or supplier of the materials being used or by reference to a suitable text or reference book. If the topic is familiar the facts, skills, strategies and concepts are listed while working through an exercise.

Continuing with the example used previously, the list of facts, skills, concepts and strategies needed to solve the problem includes the following:

Facts

Meanings for:

- pitch, rise, half span and rafter;
- definition of the ratio of two sides of a triangle;
- definition of cosine.

Skills

- finding the ratio of 2 sides by division;
- finding the cosine of an angle from calculator or tables;
- transposing equations;
- use of calculator;
- drawing mathematical diagrams;
- use of division algorithm;
- finding an estimate of the answer.

Strategies

- progressively simplifying the diagrams in stages to select a good mathematical model;
- selecting an appropriate trigonometrical ratio;
- selecting the best sequence of operations for solving an equation.

Concepts

- knowledge of roof structures as one or more triangles;
- knowledge of trigonometrical ratios for finding lengths and angles;
- knowledge of interpretation of solutions in terms of the job.

The students' prior knowledge must next be assessed to determine which facts, skills, strategies and concepts need to be developed.

PRIOR KNOWLEDGE

The fourth consideration in developing the topic plan is to take into account the prior knowledge which the students have of the new topic. There are two aspects of the prior knowledge of the students which TAFE lecturers should consider.

Firstly, some students in the class might have a sound working knowledge of the facts, skills, strategies and concepts to be taught. Teaching the topic concerned to this group would be unnecessary. It would probably cause confusion to re-teach the topic to this group particularly if the approach used for the second time differed significantly from the previous one.

Secondly, some students in the group might lack some of the basic facts, skills, strategies or concepts on which the topics to be taught are based. Hence, TAFE lecturers need an informal diagnostic procedure to identify those areas where pre-requisite facts, skills, strategies or concepts need to be developed.

In some cases, the prior teaching of mathematics to the students will have been based on memorisation of facts including terms, notation, algorithms, formulae and the like. This rote learned material can only be used when the task to which it is being applied is similar to the actual tasks performed during the learning period. Hence, TAFE lecturers have the problem of distinguishing between this superficial grasp of mathematics and the real conceptual understanding which is needed for the students to be able to apply the mathematics to their trade. This has implications for the type of diagnosis to be performed.

When school mathematics is taught by rote memorisation, the teacher introduces successively more complex problems to the students until all the situations which are going to be tested have been covered. The students memorise standard responses to the standard types of question which they are asked. If the assessment instruments also contain these standard types of question the students can obtain high test scores without having any real knowledge or understanding of the topics concerned. They will probably not be able to apply the mathematics to differing trade contexts. It follows that TAFE lecturers should not perpetuate this myth of competence by continuing to use the same standard types of question which have been used in school. If test instruments containing standard types of items are used for diagnosing the students' prior knowledge, there is the danger that recall of standard responses will be misinterpreted as indicating that the students understand the topic and can apply it. Hence, if written tests must be used, the questions should require students to explain their working fully.

It is recommended that TAFE lecturers base their decisions on:

- direct contact with individual students;
- informal discussions or other sources of information to confirm the findings of any written tests used;
- listening to oral answers to oral questions;
- observing students' oral and written attempts to solve practical, problems related to the trade;
- the students' ability to explain their understanding of a topic in their own words, with their own diagrams and symbols, or with the terms and symbols used in the trade.

A sound method for evaluating students' understanding if the group is small enough, is to set them talking about the topic concerned. The discussion will reflect the comprehensiveness and accuracy of the students' thinking.

When the prior knowledge of the students has been established the topic is planned accordingly. Understanding of the topic is then based on what the students have already learned at school.

CHOOSING AN EXAMPLE

The fifth consideration in developing the topic plan is to choose suitable examples based on the trade being taught, to use for teaching the topic.

A simple concrete example should be chosen first. It should use:

- small numbers;
- whole numbers;
- numbers, or values, which give rise to whole number answers where possible;
- common units of measurement where these help the student form the concept;
- no units of measurement if they are complex and if they can be avoided during the initial stages.

This is done as an aid to understanding, and not necessarily as an aid to calculation as most TAFE students possess calculators. For example, it is easier to understand the theory of Pythagoras using a triangle with sides of 3, 4 and 5 than using any other triangle, as the squares can be counted easily. This triangle is illustrated in Figure 15, p. 42.

If roof construction is the topic, simple concrete examples such as those in Figure 5 could be used.

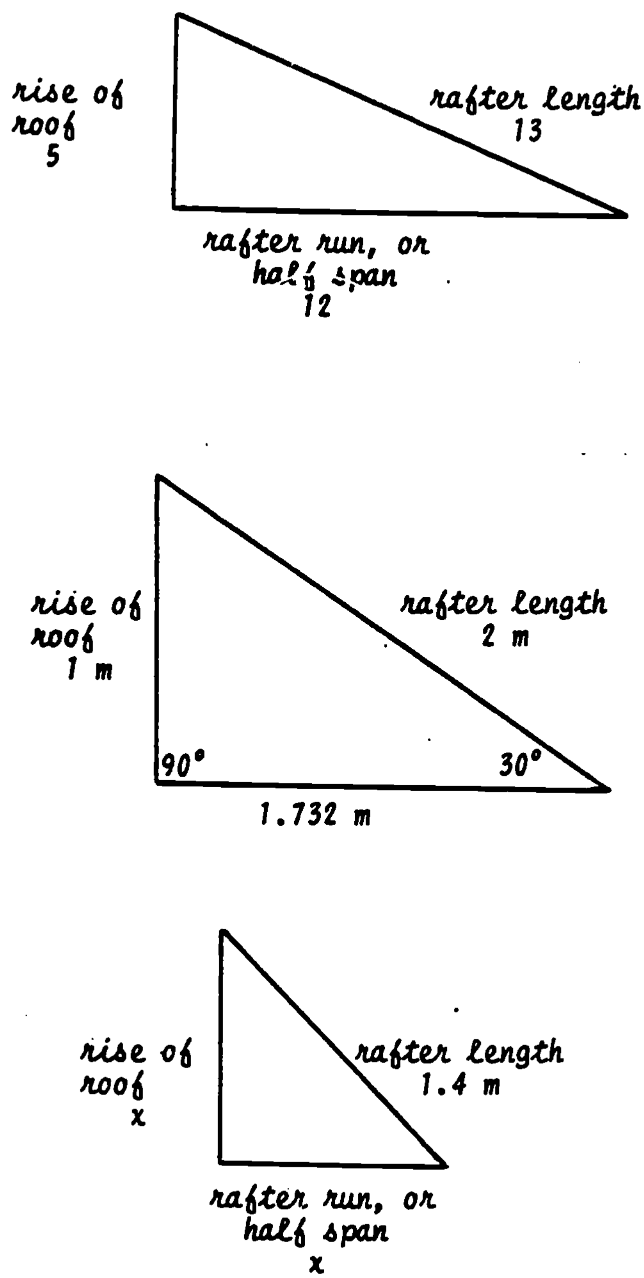


Figure 5. Simple example

LESSON PLANNING

The sixth consideration in developing the topic plan is the writing of lesson plans. The nature of these will, of course, be determined to a large extent by the teaching style of the particular TAFE lecturer. However, the number of lessons available to the lecturer, the type of resource materials available and other similar types of constraint will also have an influence on the nature of the lessons.

The structure of a typical lesson might be:

<u>Introduction</u>

Motivate the students by putting the mathematics in a suitable context.

If the mathematics is being taught in a trade class, show how the mathematics is used to solve a trade problem on the job.

If the mathematics is being taught independently of a trade, show applications in the areas of interest to the students.

<u>Prerequisite knowledge</u>

Test the students' knowledge of any material which will be needed for them to understand the new topic.

If the group is small, or can be organised into small groups, use informal assessment procedures such as observation and discussion.

If the group is large use a test instrument which measures understanding of the concepts as well as recall of standard facts.

Teach the prerequisite material if this is found to be necessary.

<u>Teach the new topic</u>

Present the new facts, skills, strategies and concepts in whatever order is logical. The best order will depend on the topic being taught and on the teaching style of the TAFE lecturer.

Present the new facts and skills first, using the
REVIEW ——► INSTRUCT ——► RECORD ——► APPLY
method described in Chapter 4.

For example:

1. Ask the students to define COS and to explain how to find a cosine.
2. Explain the material as necessary, working an example.
3. Summarise the topic for the students if they are unable to develop their own summaries.
4. Set some practice exercises.
5. Discuss cosine to evaluate. Have the students explain what they are doing.

Present the strategies using an investigation as described in Chapter 5.

For example:

1. From a plan or sketch extract the required information and write expressions for the sin cos and tan of appropriate angles.
2. Select the easiest equations to solve.
3. Discuss and record the successful methods, such as selecting equations with one unknown quantity, preferably in a numerator or as the subject of the formula.

Present the concepts using the method:

WORKSHOP EXPERIENCE ⟶ DISCUSSION AND ANALYSIS. ⟵

Re-present and review the concepts using the method:

GENERALISE ⟶ RECORD ⟶ APPLY

as described in Chapter 6.

For example:

1. Discuss a three dimensional roof scale model, or a set of photographs of partly completed houses to develop the concept of triangle usage in roofs.
2. Describe the features common to all roofs.
3. Sketch the triangles to be used for the calculations.
4. Apply the concept using plans or sketches or by making calculations related to the live work or work experience which the students have done.

Conclusion

Summarise the new topic and test the students if this is appropriate. If discussion was used extensively don't let the students guess at the correct answers. Spell out the solutions for them.

CHAPTER FOUR: TEACHING THE FACTS AND SKILLS

Before the teaching of facts and skills begins, the TAFE lecturer should determine which facts and skills the students already know and which they will need to be taught. This process is described below as a REVIEW. It is particularly important since there is a great deal of variation between TAFE students and between TAFE classes. This information about the students could be gained by informal written testing or by informal discussion and observation procedures. It is used to decide where to begin teaching. This evaluation procedure serves as a revision session for those students who are already familiar with the particular fact or skill.

If the students lack a fact or a skill, the method described in this chapter is suggested. The same teaching method is appropriate for introducing both facts and skills. If the students lack a concept or strategy, one of the methods described in the following chapters is applicable.

METHOD FOR TEACHING FACTS AND SKILLS

The following teaching process is suggested:

REVIEW ⟶ INSTRUCT ⟶ RECORD ⟶ APPLY

This should be done to establish, for each student, the basic understanding of the fact or skill.

During the INSTRUCT section of the process of teaching, the objective of the TAFE lecturer is to establish inter-connections between the new facts and skills and the students' previous experience. The new information should be explained in terms of what the students already know and understand. The methods of explaining the topic should be based on the methods previously used by the student for solving related problems, if these are known from the review.

During the RECORD section of the process, the students should be encouraged to write the new information in their own words. The TAFE lecturer should take care that the verbalisations of the students are correct as an error of expression can lead to misunderstanding and to confusion at a later time. Students may need to be given a summary of the topic particularly if the group is large. However, they should be encouraged to write their own summary first. If they can do this, they are less likely to be relying on memorisation, and more likely to have developed an understanding of the new fact or skill.

During the APPLY section of the process the fact or skill should be practised and reinforced.

Firstly, a variety of 'drill and practice' activities or exercises should be used to increase fluency with the new fact or skill. Computer programs may be used as one method of drill and practice. For example, a variety of hardware including Apple II, TRS-80, Atari, Sanyo and others is supported with 'drill and practice' software. TAFE lecturers could consider modifying these programs to make them suitable for trade students. Alternatively, new software could be developed for trade applications.

Secondly, an evaluation should be conducted by asking questions about the topic. This evaluation, as before, should be such that students' memorisation of the material is able to be distinguished from genuine understanding of the subject. Students should always be asked to explain the lessons in their own terms, either as written answers or orally. If many facts and skills are being introduced, over testing is avoided by evaluating informally.

EXAMPLES

In the roofing example used previously, some of the facts which were identified were the meanings for:

- pitch, rise half span and rafter;
- definition of the ratio of 2 sides of a triangle.

One of the skills discussed was the transposing of equations.

A method of teaching these facts and skills is presented in the following sections.

Review

The students' prior knowledge of these facts might be evaluated by asking them to sketch a roof and label the pitch, rise, half-span and rafters. If the group was small enough, a 1/10 scale building could be used instead of a sketch. Students could be asked to indicate a triangle on the 1/10 scale building and explain what is meant by the ratio of two of its sides. The TAFE lecturer should be aware that some students who do not understand that the cosine of an angle is the ratio of two sides of a triangle are able to repeat verbatim the information that cosine is adjacent over hypotenuse. Hence, this type of statement should not be accepted as evidence of understanding without further questioning of the student.

The evaluation of the students' prior knowledge of skills is similar. As an example, the skill of transposing equations is required in the roof construction context. To evaluate the students' prior knowledge, they would be asked to explain how to solve equations such as:

$$\cos 30 = \frac{1.7}{\text{rafter length}}$$

Students with a superficial grasp of the mathematics and little or no real understanding will attempt to explain how to do this using such words as 'transpose' or 'cross multiply'. They will be unable to explain how or why their process works. Students who understand equation solving will be able to explain that:

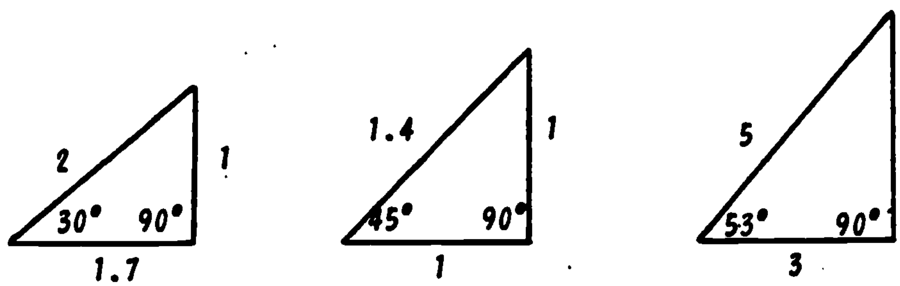
- cos 30 is a number determined by the size of the angle;
- $\frac{1.7}{\text{(rafter length)}}$ is the same number;
- multiplying both sides of the equation by the number $\frac{\text{(rafter length)}}{\cos 30}$ preserves their equality; and hence
- rafter length is $\frac{1.7}{\cos 30}$.

Instruct

The meaning of the ratio of two sides of a right angled triangle could be explained as an extension of the students' existing ideas of fractions or of division. During the INSTRUCT section of the teaching process, the students should be taught that:

- there are three commonly used ratios of pairs of sides;
- the ratios provide an alternative measure of the sizes of the angles;
- the sides may be given names to identify them.

Simple ratios such as those in Figure 6 would be calculated by the students who would then discuss the relationships between the angles and the ratios.



$\frac{1}{2} = 0.50$ $\frac{1}{1.4} = 0.71$ $\frac{4}{5} = 0.80$ using sine 30, 45 or 53

$\frac{1}{1.7} = 0.59$ $\frac{1}{1} = 1.00$ $\frac{4}{3} = 1.33$ using tangent 30, 45 or 53

$\frac{1.7}{2} = 0.85$ $\frac{1}{1.4} = 0.71$ $\frac{3}{5} = 0.60$ using cosine 30, 45 or 53

Figure 6. Relationships between angles and ratios

By discussing examples such as those in Figure 6, students are introduced to the notion that some ratios become larger and some become smaller as the particular angle of the triangle is increased.

The names used to describe the sides should be both the mathematical terms which the students know from school, and the terms used in the trade.

A suitable example to illustrate the terms which would be used in roof geometry is shown in Figure 7.

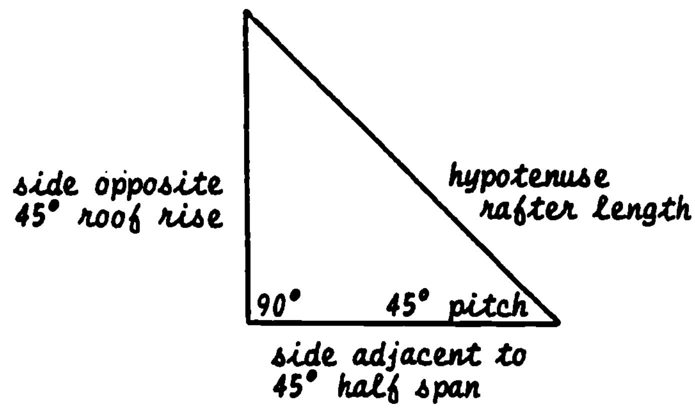


Figure 7. Terms

Record

Students should RECORD the new facts in their own words if possible. The TAFE lecturer might then provide summaries such as:

$$\sin(\text{pitch}) = \frac{\text{roof rise}}{\text{rafter length}} = \frac{\text{side opposite pitch}}{\text{hypotenuse}}$$

to enable the students to check the correctness of their own verbalisations. If students experience difficulty in expressing the ideas in words they should construct diagrams such as the one in Figure 7.

Apply

Evaluation and application using practice examples should follow, after the students have recorded the new information. Students would need to practise the writing down of ratios and the identification of the various sides including the hypotenuse and the sides opposite and adjacent to the specified angle.

Their knowledge of the new information is applied and evaluated by discussions held individually or as a group. These discussions might be based on observations made during a site visit or during the viewing of a film, film strip, video-tape or 35mm slide presentation and could involve drawing exercises or practical work.

CHAPTER FIVE: TEACHING THE STRATEGIES

One cause of concern to TAFE lecturers is the inability of some students to demonstrate the mathematical reasoning, intuition or insights necessary for them to become competent at applying their knowledge to trade contexts. These students may have mastered the mathematical notation, terminology and algorithms necessary for them to have passed school mathematics courses. Some of them have learned their mathematics by rote memorisation and still lack the necessary concepts. This is discussed in the following chapter. Other students may have developed the concepts but not learned the strategies necessary for efficient solution of problems.

METHODS FOR TEACHING THE STRATEGIES

Informal evaluation methods, such as asking students to work through an example, or oral questioning, should be used to determine the students' prior knowledge of the strategies.

If the students are familiar with the strategies, they will need only a brief review. The the method of providing them with a variety of practical exercises should be used. The strategies needed to solve practical problems are then explained as they are encountered by the students. Hence, students should be provided with a variety of practical exercises. The range of exercises should be such that all the necessary strategies are illustrated.

If the students are not familiar with the strategies the alternative procedure of using a mathematical investigation is recommended. The students should be encouraged to carry out a range of mathematical investigations to determine which skills lead to successful solutions and which do not. This approach is more theoretical and time consuming than the first. If it is used the students need at least one practical trade exercise to assist them in the transfer of their newly learned strategies to the contexts in which they are to be applied.

EXAMPLES

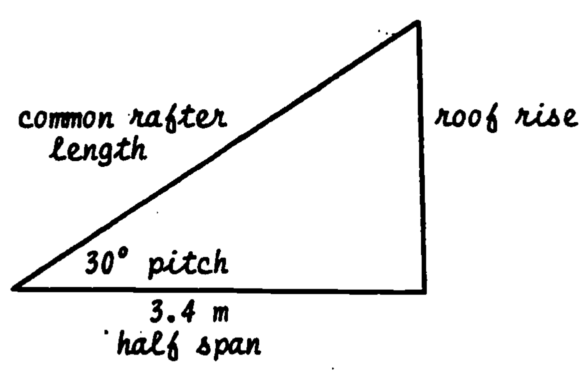
In the roofing example used previsouly, one of the strategies which was identified is the selecting of cosine rather than sine or tangent as the appropriate trigonometrical ratio for finding the length of a common rafter. Another strategy used is the method of selecting the tan of the pitch to use when finding the roof rise.

If these two strategies are to be taught by the method of providing the students with a variety of practical exercises, an assignment sheet could be prepared. Students would be required to calculate the roof rises and the common rafter lengths from several plans. Each would have a different span and pitch of roof specified. The first example on the assignment sheet might be worked in class to illustrate the method to be used. Students would then practise the strategies using the remaining exercises.

If the two strategies are to be taught by a mathematical investigation, the assignment sheet might have only one plan. For example a span of 6.8 metres and a pitch of 30 degrees would be specified. Students would be required to:

- model the situation as a triangle;
- write equations using as many trigonometrical ratios as they know;
- substitute the known quantities into the equation;
- use their answers to explain how to select the best ratio for finding the roof rise and rafter length.

A typical student's answer might be as shown in Figure 8.



From the diagram, it can be seen that:

$$\sin 30 = \frac{\text{roof rise}}{\text{rafter length}}$$

$$\cos 30 = \frac{\text{half span}}{\text{rafter length}}$$

$$\tan 30 = \frac{\text{roof rise}}{\text{half span}}$$

and that:

$$0.50 = \frac{\text{roof rise}}{\text{rafter length}}$$

$$0.85 = \frac{3.4}{\text{rafter length}}$$

$$0.59 = \frac{\text{roof rise}}{3.4}$$

From this set of equations it can be seen that the second equation which uses the cosine ratio can be easily used to find the rafter length. It can also be seen that the third equation which uses the tangent ratio can be easily used to find the roof rise.

Figure 8. Student answer

REVIEW AND CONSOLIDATION

After such an investigation is made it is necessary to relate the strategy to the trade context. Students should be required to work some trade related exercises which require a knowledge of the strategy.

It is usually necessary to teach strategies after students are familiar with the facts, skills and concepts. Hence the teaching of strategies provides an opportunity for the TAFE lecturer to provide a review and overview of the entire topic. The use of trade examples also provides the opportunity to review the trade terminology and to relate the mathematics back to this context. A summary of the whole topic helps the students to remember the strategies they have learned.

CHAPTER SIX: TEACHING THE CONCEPTS

As with the teaching of facts, skills and strategies, the first step in the teaching of concepts is to determine what the students already know about the new concept. If the mathematical concept is being taught in the context of the particular trade, the TAFE lecturer will be aware of the students' knowledge of technical or trade terms relating to that concept. Otherwise the students' knowledge of the context in which the concept is to be taught should be evaluated.

Informal evaluation strategies are suggested rather than formal tests. Activities such as discussion, observation of the students' performance on relevant practical tasks, and oral questioning should be used to determine the students' prior knowledge of the concept. TAFE lecturers should be careful not to interpret accurate recall of notation, definitions and algorithms as indications that the student has developed an understanding of the required concepts. While time permits the students should be required to explain the reasons for and the implications of every statement that they make, to ensure that they are not repeating verbatim, without understanding, some of the formulae they have memorised.

METHOD FOR TEACHING THE CONCEPT

If the basic understanding of the concept is lacking, the teaching process suggested is:

WORKSHOP EXPERIENCE ⇄ DISCUSSION & ANALYSIS

→ GENERALISE → RECORD → APPLY

The first section of the process:

WORKSHOP EXPERIENCE ⇄ DISCUSSION & ANALYSIS

is concerned with developing the new concept.

The second section of the process:

GENERALISE ——► RECORD ——► APPLY

is concerned with representing and reinforcing the concept.

When the TAFE lecturer finds that the students are familiar with the concept required, the first section of the model may be omitted to avoid unnecessary repetition which could bore the students and waste valuable time. The application of the teaching process is described in the following section.

DEVELOPING THE CONCEPT

The authors of this paper believe that where concept development is the objective, concrete experiences should lead to abstractions, a study of particular cases should lead to generalisations, and the students' existing knowledge should be extended to enable them to understand new situations.

WORKSHOP EXPERIENCE

During the WORKSHOP EXPERIENCE section of the process the students should be provided with practical exercises which illustrate the concept and which use as many of the students' senses as is practicable. This practical experience provides them with concrete examples on which to base their thinking about the new concept.

DISCUSSION AND ANALYSIS

When the new concept has been presented in this way, the DISCUSSION AND ANALYSIS section of the process is used to provide immediate feedback on the meaning of the practical experience, and to introduce the language and terminology associated with the new concept. Discussion is very important in the learning of many mathematical concepts because of the essentially verbal nature of the concepts themselves. A variety of problems and exercises should be used in the discussions and analysis sessions, and these should relate to the trade. The sequence of workshop experiences followed by discussion and analysis of the observations is repeated as many times as is necessary for the students to be able to specify and quantify the relationships involved.

GENERALISE REPRESENTING AND REINFORCING THE CONCEPT

After the students have developed an intuitive idea of the relationships involved through experiences involving a number of particular cases in the workshop, the concept should be stated in general terms. This is the GENERALISATION section of the process. From a knowledge of the particular workshop experiments the students should develop an understanding of the rules, laws, algorithms, and principles involved. Discussion activities should be used, although TAFE lecturers should not hesitate to provide the answers, particularly for the more abstract concepts. TAFE lecturers should act as interpreters and translators of the most abstract concepts into terms which are meaningful and familiar to the students. Students should be required to explain the rule, law, algorithm or principle in their own words in addition to being required to explain that same rule, law, algorithm or principle using correct trade terminology and technical notation.

The units of measurement should be considered at this stage. If they were omitted to simplify the development of the concept they should now be introduced and related to the practical context discussed above. If they were not omitted previously, they should be discussed again and related to the particular concrete examples being used.

RECORD

During the RECORDING section of the process, students are encouraged to represent the concept in their own words, or by using symbols. A formula or rule is the result. This verbal or symbolic representation of the concept is done orally or in written form.

Slower working students may need to be given a summary of the topic including the correct terminology and units, but it is preferable for the TAFE lecturer to refine students' work and correct any errors in the students' efforts before presenting them with a summary. Particular care should be taken to check that the verbalisations made by the students are correct to prevent difficulties occurring at the application stage.

APPLY

During the APPLICATION section of the process, specific examples from the trade situation should be worked through by the TAFE lecturer to give the students an overview of the whole scene, and some insight into the use of the concept. This application section can also be used to evaluate the students' grasp of the concept. When students are applying a new concept, they should be required to explain the reasons for their actions at every opportunity. The TAFE lecturer should take steps to create these opportunities as frequently as is practicable. In this fashion, the TAFE lecturer can ensure that the students have formed a concept and are not simply memorising the material without the necessary understanding of what they are doing.

EXAMPLES

In the roofing example used previously, one of the concepts which was identified is knowledge of trigonometrical ratios for finding lengths and angles. It is assumed that the prerequisite facts and skills concerned with the use of the trigonometry have been taught.

The informal evaluation techniques which were described previously should be used to determine the students' prior knowledge of the concept of trigonometrical ratio. During this process, the students should be required to explain the reasons for their answers. This approach is easier than the alternative which is to contruct a test out of items which are all unfamiliar to all of the students.

WORKSHOP EXPERIENCE, DISCUSSION AND ANALYSIS

If the TAFE lecturer considers that the basic understanding of the concept is lacking, the WORKSHOP EXPERIENCE, DISCUSSION & ANALYSIS method should be used. This is done using practical exercises and discussion. Model buildings, photographs or plans might be used. Students could be given hands-on experiences, for example in locating, marking and measuring the triangles used on a scale model of a building, on a plan, or on a photograph of a partially constructed roof. The form of the roof might be constructed by attaching string, or masking tape, to a cube-shaped frame such as the one illustrated in Figure 9.

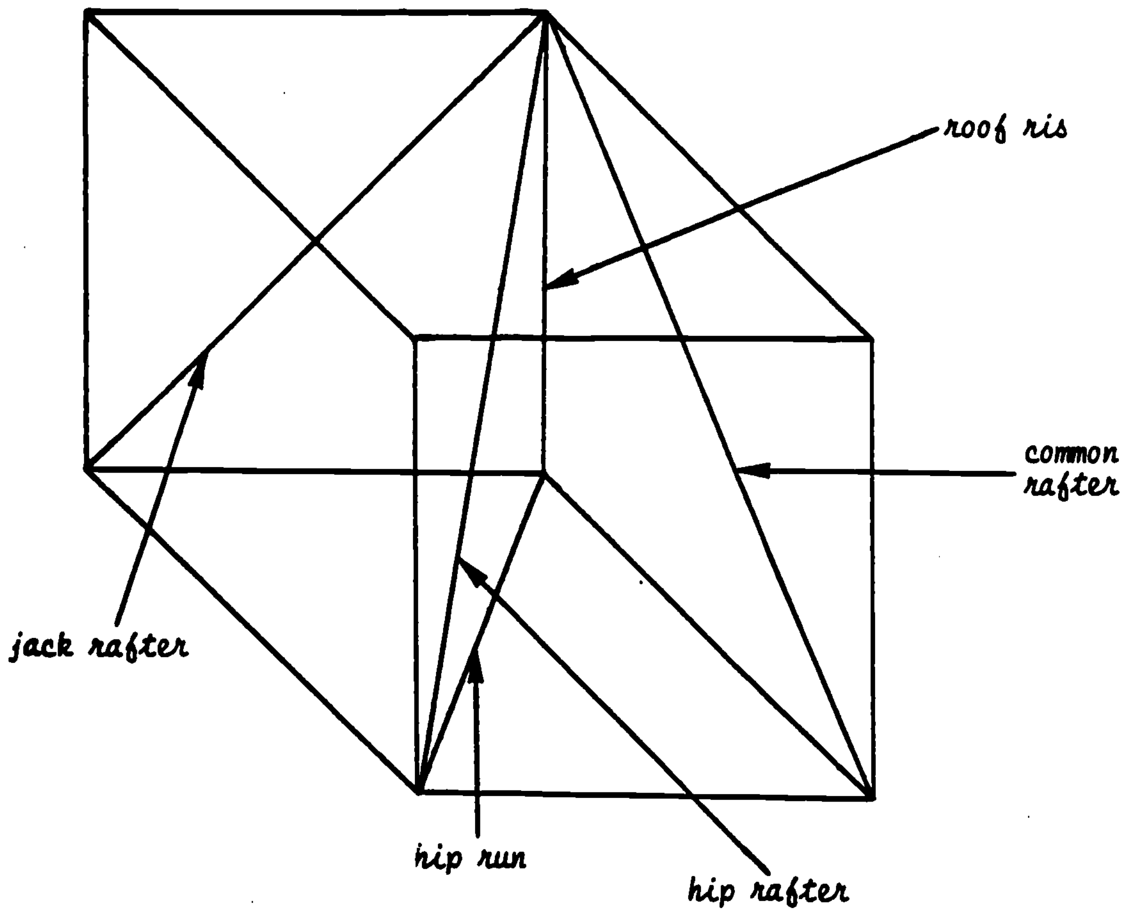


Figure 9. Wooden cube

The concrete work and discussions are continued until each student is able to visualise and indicate the geometrical shapes within the roof which they are going to use, and to describe which lengths and angles they know and which must be calculated.

GENERALISE

From the understanding of the geometry which they have developed using resources such as models, photographs and plans, the students should illustrate on a perspective drawing of a roof, a GENERALISATION of the particular structures involved, and of the information known and required.

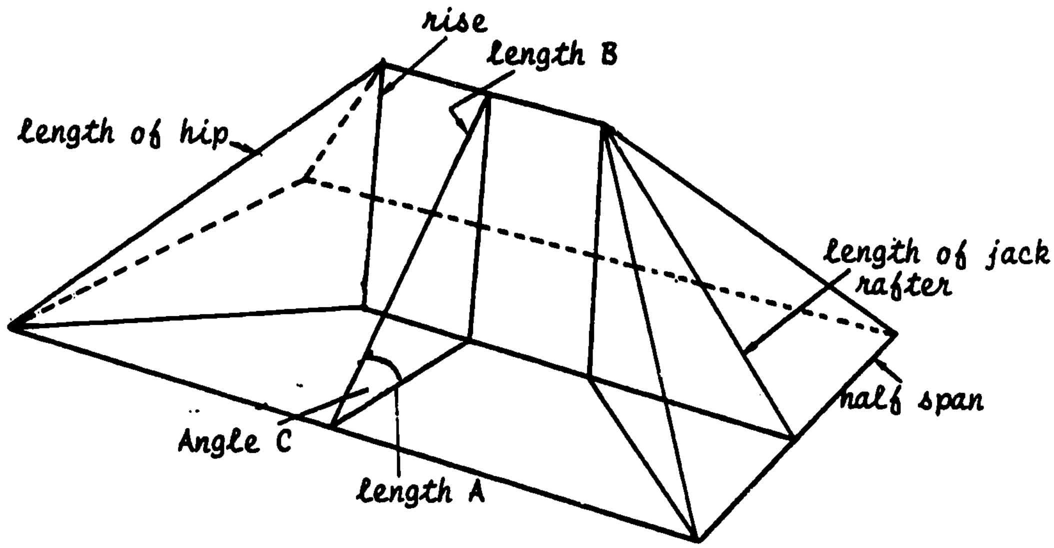


Figure 10. Roof diagram

On diagrams, such as Figure 10 for instance, the students should be able to indicate that length 'A' divided by length 'B' gives the cosine of the angle 'C'. They should be able to explain that angle C and length A are known from the plan or specifications, and that as a result this equation has only one unknown quantity. Hence, the length B may be found by solving. If they can explain their working in this fashion they will be able to transfer their knowledge to other situations.

RECORD

Students should RECORD all the trigonometrical ratios they have learned. For example, a student might include in his notes verbalisations such as:

- tan(pitch) = rise/half span;
- sin(pitch) = rise/length of rafter.

It is important to check that the notes made by individual students are both correct and understood by the students.

APPLY

In the APPLICATION stage of the teaching process a complete example should be worked both by the TAFE lecturer and by individual students. This serves also as an overview of the whole concept.

The roofing example is discussed in greater detail in the following chapter.

The material in this chapter will be of interest to two groups of teachers:

1. Those who teach or have an interest in roofing geometry;

2. Those seeking further explanation of the ideas expressed in Chapters 1 to 6.

It is suggested that teachers read whichever of Chapters 7, 8 and 9 most closely resembles the subject they teach.

When measuring roofing work there are three lengths which are commonly required. These are the length of the common rafters, the rise of the roof and the length of the hip rafters. The length of the common rafters is used, for example, to calculate the area of tiles on the roof. The rise of the roof is used to calculate the length of the hip rafters. Students frequently encounter problems in visualising the position of the hip rafter because it is inclined to the horizontal at an angle less than the pitch of the roof. In this chapter, the modelling, solving and interpretation stages which should be considered when teaching this topic are identified. The mathematical facts, skills, strategies and concepts are identified and discussed. One method for teaching the mathematics in trade context is outlined.

CONTEXT

It should be remembered that the method presented here is not the only possibility. However, its use is advocated in the belief that methods based on concept formation give rise to better results than methods based on memorisation. Well developed concepts, for example, will enable the students to apply their knowledge to the wide variety of similar situations which they will encounter in practising their trade.

It should also be remembered that if a different approach to the trade topic is used, different mathematical facts, skills, strategies and concepts will be involved. The approach used and hence the facts, skills, strategies and concepts needed should be decided after consideration of the mathematical background of the students. An approach based on mathematical ideas which are familiar to the students will then be chosen. Other factors which should be considered include the locally preferred methods, the stage which the students have reached in their course, and the ways the students have been taught the mathematics in previous courses.

ANALYSIS OF THE TASK

The modelling solving and interpreting stages which the students are to follow should be specified next. An example follows.

MODELLING

In a case where the students are not familiar with the construction of a pitched roof, the roof might be represented as a perspective as shown in Figure 11.

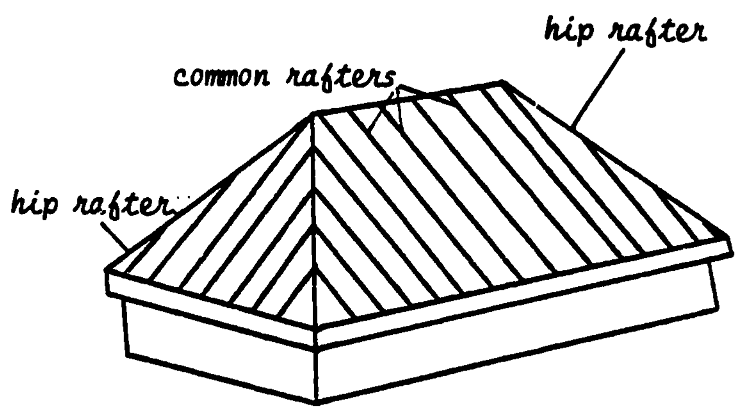


Figure 11. Roof perspective

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A section through the roof as illustrated in Figure 12, is important to assist the students in interpreting the solutions they will obtain from their calculations.

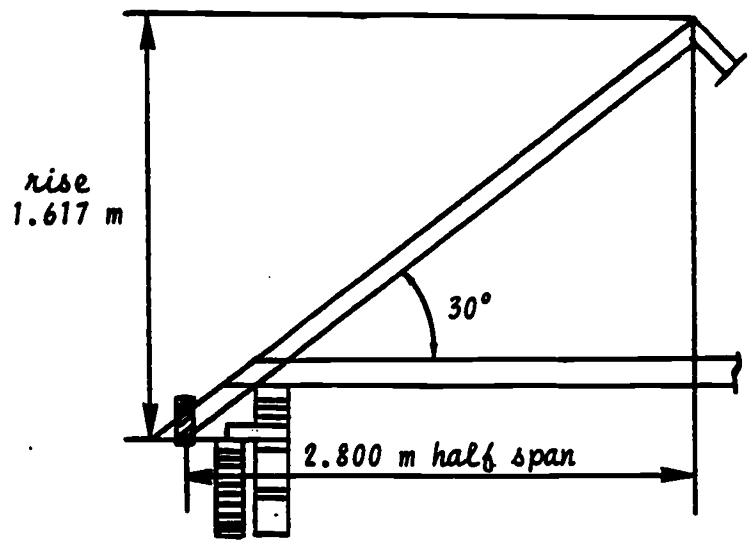


Figure 12. Roof section

Figures 13 and 14 show a plan of the roof and a diagram indicating a triangle in a vertical plane through the hip rafter, both of which are useful for indicating the calculations which are to be made.

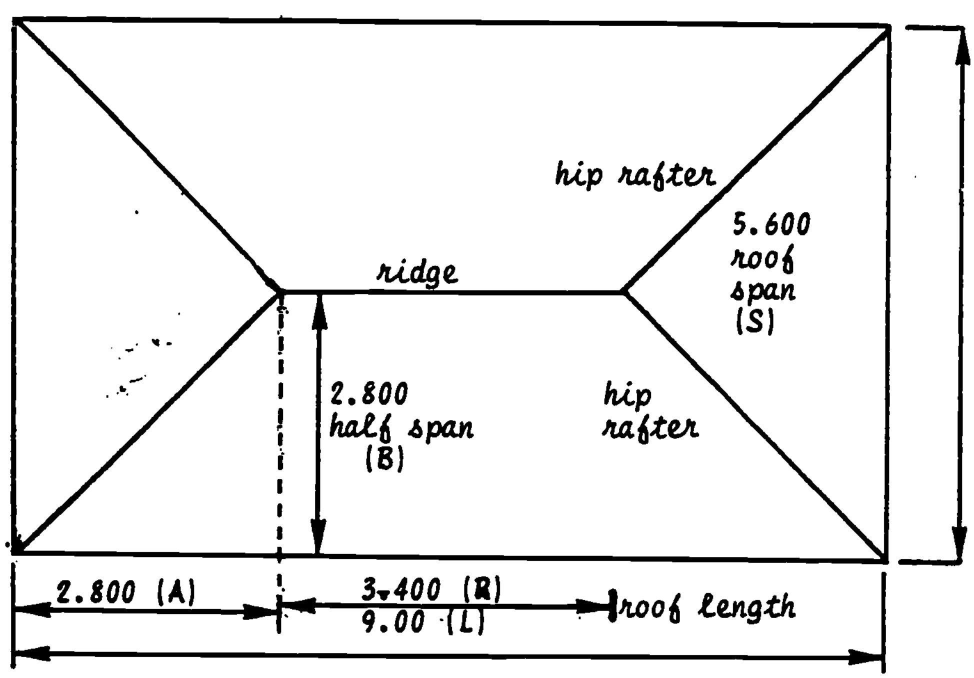


Figure 13. Plan

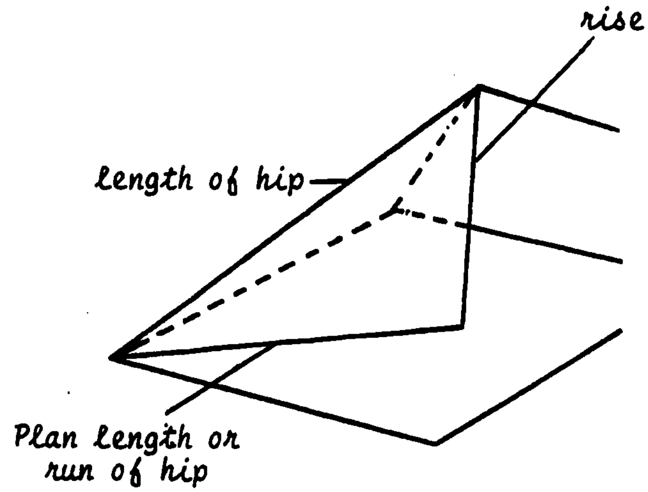
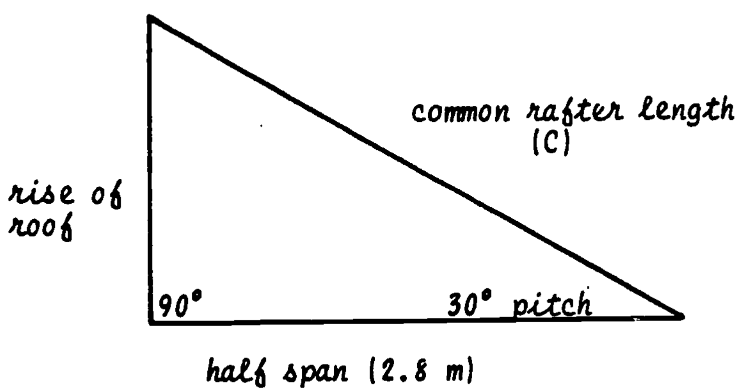


Figure 14. Plane of the hip

SOLVING

One of the acceptable methods which the students might use to calculate the lengths is described below.

__Length of common rafter__



$$\cos \text{(pitch)} = \frac{\text{half span}}{\text{common rafter length}}$$

$$\text{so } \cos 30 = \frac{2.8}{\text{common rafter length}}$$

$$\text{so common rafter length} = \frac{2.8}{\cos 30}$$

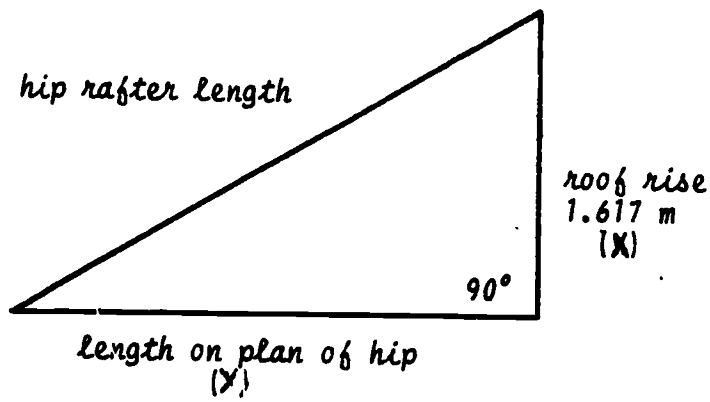
$$= 3.233 \text{ m}$$

Rise of roof

$$\tan(\text{pitch}) = \frac{\text{rise of roof}}{\text{half span}}$$

$$\begin{aligned}\text{so rise of roof} &= \text{half span} \times \tan 30 \\ &= 2.8 \times \tan 30 \\ &= 1.617 \text{ m}\end{aligned}$$

Length of hip rafter



$$\begin{aligned}\text{length on plan of hip} &= \sqrt{(A^2 + B^2)} \\ &= \sqrt{(7.84 + 7.84)} \\ &= \sqrt{(15.68)} \\ &= 3.960 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{length of hip rafter} &= \sqrt{(X^2 + Y^2)} \\ &= \sqrt{(1.617^2 + 3.960^2)} \\ &= \sqrt{(18.296)} \\ &= 4.277 \text{ m}\end{aligned}$$

Area of tiles

$$\begin{aligned}\text{area} &= C \times (S + R + L) \\ &= 3.233 \times (5.6 + 3.4 + 9.0) \\ &= 58.2 \text{ sq. metres.}\end{aligned}$$

INTERPRETING

The first important facet of the interpretation stage is to recognise the limitations of the model. From a careful consideration of Figure 4 it can be seen that the calculated length does not take into account timber wasted in cutting the foot of the rafter, nor does it allow for the reduction in the rafter length by half the ridge thickness. If there is a need to calculate the dimensions of the finished rafter, a different mathematical model, based on a diagram such as Figure 4 should be used. However, it is not necessary to calculate this as it is usually more convenient to use methods based on direct measurement for the marking and cutting process.

The other important facet of the interpretation stage is the estimation of the answer. This enables some mathematical errors to be detected. It gives the student confidence that the answer is reasonable. For example, the roof area may be estimated. From the diagrams, it can be seen that the plan of the roof is 9m long by 6m wide approximately, so 58 square metres is a reasonable roof area. The current fashion is to have the students estimate before they calculate, hoping that they will not omit the estimation. The risk in this is that they will not see any need to calculate.

ANALYSIS OF THE MATHEMATICS

The mathematical facts, skills, strategies and concepts which the students will need should be listed next. An example follows.

<u>Facts</u>

The facts which the students are to learn include:

- the definitions of the terms, for example 'pitch';
- the definitions of all trigonometric ratios used, for example sine, cosine and tangent;
- the definition of the ratio of two lengths;
- the names used to describe the sides of a triangle relative to a particular angle;
- the formula of Pythagoras' theorem.

Skills

The skills which the students are to develop include:

- finding the ratio of two lengths;
- calculating the sine, cosine or tangent of an angle from a triangle;
- finding the sine, cosine or tangent of an angle with tables or an electronic calculator;
- algebraic operations for solving an equation;
- finding squares and square roots with tables or calculators;
- solving triangles using Pythagoras' theorem;
- calculating by hand or with a calculator;
- substituting particular values in an equation;
- finding an estimate of the approximate answer.

Strategies

The strategies which the students are to develop include:

- progressively simplifying the diagrams of the job to select a suitable mathematical model;
- selecting Pythagoras' theorem if 2 sides are easily obtained by measurements or from a plan;
- selecting a trigonometrical method if an angle and one side are known or easily obtained;
- selecting the easiest way to solve an equation;
- selecting the method of arriving at the most accurate estimate of the answer.

Concepts

There are several concepts involved. These include the bodies of knowledge about:

- roof geometry
- arithmetic
- finding angles and sides of triangles
- areas of surfaces
- technical drawing and sketching;
- interpreting solutions in terms of the job.

LESSON PLANNING

The roofing topic would probably be taught as a sequence of lessons. In each lesson, a short introductory example relevant to the trade could be used to place the mathematics in a context which the students will see as being of some use to them. The example should illustrate where the mathematics is used in trade applications. If the subject matter for one lesson was the use of Pythagoras' theorem to find the length of the hip rafter, the example used might be based on the relationship: $(\text{length on plan of hip})^2 + (\text{rise})^2 = (\text{length of hip})^2$

REVIEW

If the group was small, the students' prior knowledge could be evaluated by oral questioning. Students would be asked:

- to find the squares and square roots of numbers using a calculator, using tables, or by whatever method is preferred;
- to explain what squares and square roots mean using a geometrical example;
- to state the theorem of Pythagoras.

This material would be taught if it was found that the students did not understand what they were doing.

FACTS AND SKILLS

Firstly the fact of Pythagoras' theorem might be presented.

$$c^2 = a^2 + b^2$$

$$\text{or} \quad c = \sqrt{(a^2 + b^2)}$$

Secondly, the skills involved in calculating c given values for a and b would be learned. For example, students might practice calculations such as:

$$c = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = 5$$

The INSTRUCT, RECORD, APPLY methodology would be used. Each item which is not familiar to the students would be explained in terms of their prior knowledge, recorded in their own terms and practised. The evaluation conducted for each fact or skill should be designed to detect students memorising facts without understanding them.

CONCEPTS

The concepts listed above would be presented next using the methodology

WORKSHOP EXPERIENCE ⇄ DISCUSSION & ANALYSIS;

and represented and reinforced using the methodology

GENERALISE ⟶ RECORD ⟶ APPLY.

Students who need concept development activities might be presented with a variety of learning resources. For example, Pythagoras' theorem would be discussed and explained in terms familiar to the students initially using concrete examples such as that illustrated in Figure 15. Alternatively site visits or workshop equipment may be used to assist in the development of concepts such as knowledge of roof geometry.

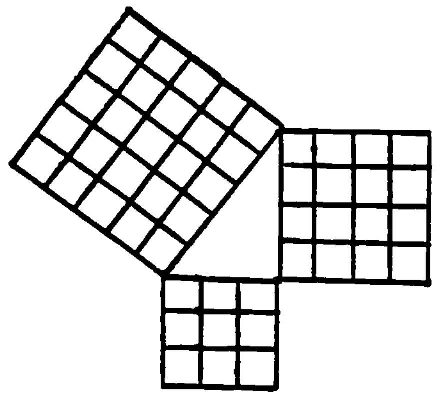


Figure 15. Concrete example

An excursion or site visit would be a suitable activity for a pre-vocational group to familiarise students with various general or specific features of roofing. For example, a walk around the local district to observe roof shapes and styles might be an appropriate beginning activity. The same, or a later excursion, might be used to observe a variety of roofing materials, claddings and accessories such as gutters, spouting, down-pipes, facia boards, ridging and cappings.

Subject to the availability of suitable venues, visits would be arranged to sites where:

- work is in progress in roof construction;
- work is in progress on re-covering an old roof;
- preparations are under way for a roof construction job;
- unusual roof constructions or uncommon materials have been used;
- specific features of roofs are clearly demonstrated.

Site visit resources such as excursion notes, on-site training manuals, checklists and evaluation sheets should be used to prepare the students for the excursion, and to help them to record and systematise the information gained during the visit and afterwards.

The TAFE lecturer should use classroom or workshop devices such as:

- film, slide packs, video tapes or photographs depicting a variety of roofing styles, approaches to construction or materials;
- a mixture of global views and close-up photography;
- enlarged technical drawings;
- model roof constructions built to scale in wood, aluminium, steel or plastic;
- the Wooden Cube with coloured strings and name tags;
- refinements of the Wooden Cube with interlocking rods and blocks to show cable ends, hips and valleys;
- computer simulations and graphics aligned to the technical drawings;
- calculators;
- drawing instruments.

Each experience session should be followed by a discussion session or by the provision of a solution to the exercise involved.

When students are familiar with the concept of applying Pythagoras' theorem to triangles in a roof they should be required to complete assignments such as:

- illustrating the applications of the mathematics on a perspective drawing;
- calculating the required lengths;
- constructing a scale size structure of a roof.

STRATEGIES

The <u>strategies</u> could be taught by working realistic examples. These should review the whole topic as well as illustrate the strategies used. Students might be encouraged, during this review period, to investigate alternative methods of arriving at an answer. In this way, students should gain an insight into their strengths and weaknesses and be able to select the most favourable strategy for solving problems.

For example, the strategy of selecting Pythagoras' theorem rather than a trigonometrical formula might be presented. Students would be given an assignment sheet containing several examples, only half of which could be solved easily using Pythagoras' theorem.

Following such an investigation, students would work a complete example starting with information given on a plan or perspective drawing. This might be used for assessment if it was considered appropriate to assess at this particular point in the course.

CHAPTER EIGHT: ELECTRICAL APPLICATIONS

The material in this chapter will be of interest to two groups of teachers:

1. Those who teach or have an interest in electrical applications;

2. Those seeking further explanation of the ideas expressed in Chapters 1 to 6.

It is suggested that teachers read whichever of Chapters 7, 8 or 9 most closely resembles the subject they teach.

In this chapter, the problem of calculating the current flowing in a parallel circuit is considered. The modelling, solving and interpreting stages are identified. The mathematical facts, skills, strategies and concepts are identified and discussed. One method of teaching the mathematics involved in the topic is outlined.

CONTEXT

The examples used should be related to the area of interest of the students. Typical voltages and types of circuits should be used. Applications which are likely to be encountered by the students in their practical work should be selected.

ANALYSIS OF THE TASK

The modelling, solving and interpreting stages which the students are to follow should be specified next. In the following sections an example is used to illustrate how this may be achieved.

MODELLING

The electrical situation is modelled as a circuit diagram. If an additional step is required, it may be modelled first as a system of plumbing as shown in Figure 16.

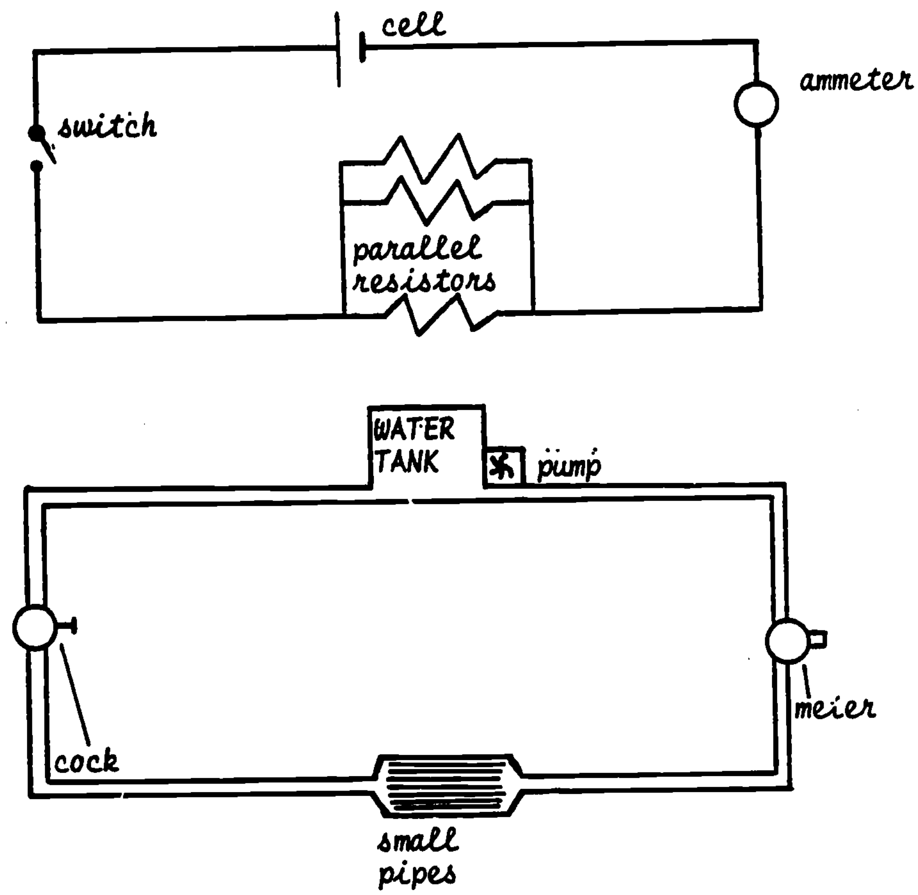
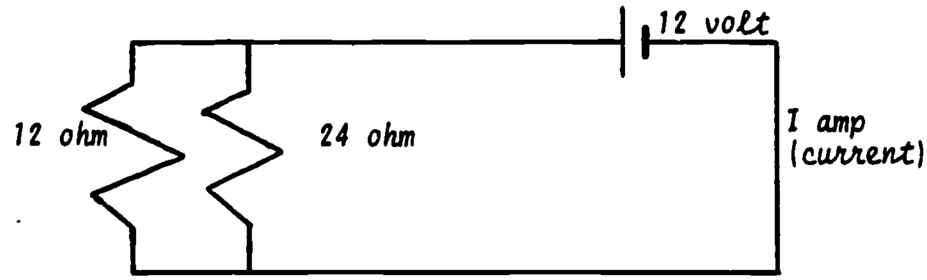


Figure 16. Liquid model of circuit

SOLVING

If the context for the problem is a 12 volt system containing two filaments in parallel, a suitable method of finding the current flowing might be as follows.



$$I = V \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

where:

I is the current drawn when both filaments are working;
V is the voltage, in this case 12 volts; and
R_1, R_2 are the resistances of the two filaments.

$$I = 12 \left(\frac{1}{12} + \frac{1}{24} \right)$$

$$= 1.5 \quad \text{(by calculator, or division and addition)}$$

Alternatively, students might use the strategy of adding the currents flowing in the two branches. For example:

$$I = I1 + I2$$

so $$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = \frac{12}{12} + \frac{12}{24}$$

$$I = 1.5$$

These two strategies are mathematically equivalent since:

$$V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{V}{R_1} + \frac{V}{R_2}$$

INTERPRETING

To relate this answer to the practical situation, the students must be aware of the reasons for wanting to calculate the current flowing in the circuit. They might draw conclusions about the type of wire required for the circuit, the size of fuse which should be fitted, or the type of switch to be used.

The second part of the interpreting process is the estimation of the answer to check the accuracy of the calculations. From the definitions of resistance and of the unit (the ohm), the students should be able to estimate the current drawn by one motor to be 1 amp, and that drawn by the other to be 0.5 amp. Hence, 1.5 amps would be accepted as a reasonable answer.

ANALYSIS OF THE MATHEMATICS

The facts, skills, strategies and concepts which the students need are listed next. An example follows:

Facts

The mathematical facts which the students are to learn include:

- units, such as ampere, volt, and ohm;
- notation used, such as the 'is proportional to sign, and 7/2 for the reciprocal of 2/7.

Skills

The mathematical skills which students are to develop include:

- transposing;
- drawing circuit diagrams;
- arithmetical processes, particularly the calculation of reciprocals.

Strategies

The strategies which might be used to solve the problem include:

- using a circuit diagram to assist in selecting the best method of solving;
- adding the currents flowing in the branches;
- calculating the combined resistance of the two motors as the first step.

Concepts

The concepts which are involved include:

- knowledge about direct proportion;
- knowledge about circuits, including the use of a circuit diagram as a model of the circuit;
- knowledge about DC;
- knowledge about resistors.

LESSON PLANNING

This topic might be taught in a single session or in a sequence of lessons depending on the background and ability of the students. The topic should be introduced in the context of the particular trade concerned. For example, if an automotive electrical course is the context, an example from a motor vehicle should be used and the topic should be approached using a 12 volt direct current circuit.

Review

In a small group, the students' prior knowledge would be evaluated by oral questioning. For a larger group, where a test was used, students' answers would be further investigated by oral questioning in cases where there was a possibility that the students might not really understand the topic.

Any of the material which the students do not understand from school should be presented. Care should be taken to present the information in a context which will enable the students to appreciate how it is used in the trade.

Facts and skills

The facts and skills which are not known would be presented first. For example, students might need discussion of the symbols used in trade areas which are not taught in school. The skill of using these to draw a circuit diagram follows logically. The REVIEW ⟶ INSTRUCT ⟶ RECORD ⟶ APPLY method is used.

Concepts

The concepts would be taught next. The concept development process might consist of alternate WORKSHOP EXPERIENCE and DISCUSSION. To investigate the concept of DC, for example, students would be required to investigate first the direct proportionality between EMF and current using equipment such as that described in Figure 17.

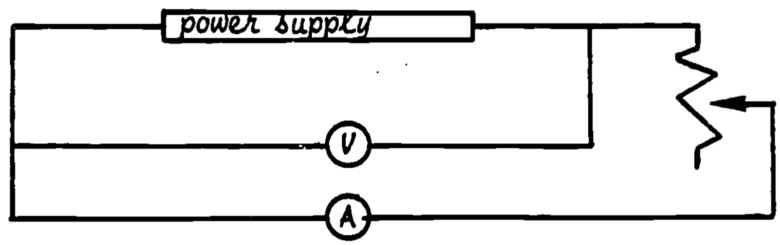


Figure 17. Workshop equipment

If time permitted exercises might include:

- increase the EMF, observe the current, draw a graph;
- double the EMF, observe the change in current;
- observe the current for three different values of EMF, calculate V/I.

In the discussion sessions a concrete basis for the understanding of the relationship is developed. Students should intuitively understand that:

- doubling V causes I to double;
- hence V/I is always the same value;
- the fact that this value is called the resistance.

When this concept has been developed, it may be GENERALISED and RECORDED as:

	V is directly proportional to I;
that is	$V \propto I$
hence	V/I is a constant
so	$V/I = R$
and	$V = I \times R$
or	$V = R \times I$.

During the RECORDING of this relationship, the units should be introduced and discussed. In this particular topic, the units have been considered from the beginning, as the voltmeter and ammeter were used in the workshop sessions. Simple examples to consolidate the understanding of Ohm's law could be worked at this point. The TAFE lecturer should design the exercises carefully so that the students must explain at least one problem fully in their own words. Technical terms could be used where these help keep the explanations, given by the students, in context.

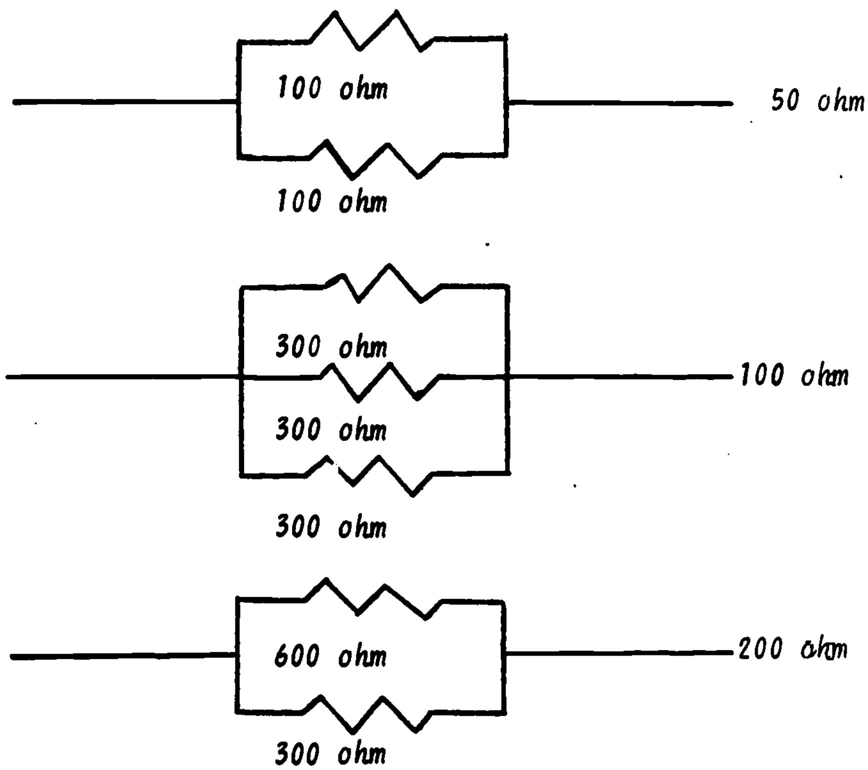


Figure 18. Parallel resistors

When the concept of resistance is being developed, students might be required to investigate the properties of various combinations of resistors using a multi-meter.

The WORKSHOP EXPERIENCE ⇄ DISCUSSION & ANALYSIS method is used.

For example, by finding the combined resistance of components such as these shown in Figure 18 students can conclude that:

- the combined resistance is less than either of the single resistances;
- the combined resistance of 3 equal components is 1/3 of the resistance of each component;

- the combined resistance is found by using reciprocals, for example:

$$\frac{1}{300} + \frac{1}{600} = \frac{1}{200}$$

This can be generalised and recorded as the formula:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \frac{1}{R_{30}}$$

The students would then apply this formula to some practical exercises.

Strategies

The <u>strategies</u> would be discussed next. The students might be encouraged to make the connection between parallel resistance and Ohm's law by theoretical investigations, such as the following, if their background knowledge was sufficient.

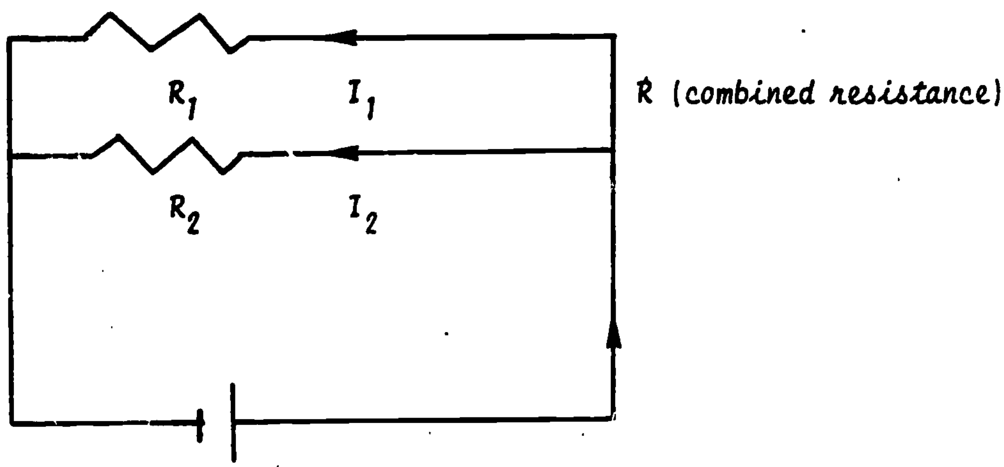


Figure 19. Combined resistors

From the curcuit shown in Figure 19 it can be seen that:

$$I = I_1 + I_2$$

so, using Ohm's law $$I = \frac{V}{R_1} + \frac{V}{R_2}$$

and dividing by V $$\frac{I}{V} = \frac{1}{R_1} + \frac{1}{R_2}$$

or $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

From a mathematical investigation, such as this, the students can understand the strategy of first finding the combined resistance of the parallel section of the circuit and then using Ohm's law to find the current.

After they have done this they will understand the formula $I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ which was used for calculating the current.

Informal evaluation procedures should be used to test the students' understanding of the idea that this formula is a statement of Ohm's law. A complete exercise should be worked to illustrate the use of the strategy, and the students should then have adequate practice at solving problems relevant to their trade.

The material in this chapter will be of interest to two groups of teachers:

1. Those who teach or have an interest in foundry;

2. Those seeking further explanation of the ideas expressed in Chapters 1 to 6.

It is suggested that teachers read whichever of Chapters 7, 8 or 9 most closely resembles the subject they teach.

CONTEXT

Metallurgists may be required to calculate the amounts of various metals and alloys which have to be melted to produce an alloy of specified composition. For example, ferro-manganese contains 70% manganese and 7% carbon. Pig iron contains 4% carbon. These are to be added to 5000 kg of mild steel to raise its carbon content from 0.2% to 0.3% and to raise its manganese content from 0.4% to 0.7%. The amounts of ferro-manganese and pig iron which must be added are to be calculated. For this application, the modelling, solving and interpeting stages are identified. The mathematical facts, skills, strategies and concepts are identified and discussed. One method for teaching the topic is outlined.

ANALYSIS OF THE TASK

The modelling, solving and interpreting stages which the students are to follow should be specified next. An example follows.

MODELLING

Students might use the method of choosing variables to represent the number of kilograms of ferro-manganese and pig iron required. For example:

- x is the number of kilograms of ferro-manganese required;
- y is the number of kilograms of pig iron required.

The relationships between these variables then provide a mathematical model of the problem. For example, students might make the assumption that the amount of carbon remains constant during melting, so:

(0.2% of 50000kg) + (7% of x kg) + (4% of y kg) = 0.3% of (5000 + x + y) kg

(0.4% of 5000kg) + (70% of x kg) = 0.7% of (5000 + x + y) kg

The mathematical model of the situation can then be seen to be the pair of equations:

$$(0.002 \times 5000) + 0.07x + 0.04y = 0.003\ (5000 + x + y)$$
$$(0.004 \times 5000) + 0.70x = 0.007\ (5000 + x + y)$$

SOLVING

One strategy which students might use to solve this pair of equations simultaneously is to eliminate one of the variables after writing the equations in the same form.

The equations become:

$0.067x + 0.037y = 5$ using the distributive and cancellation principles

$0.693x - 0.007y = 15$

or $0.067x + 0.037y = 5$
$3.663x - 0.037y = 79.286$

so $3.730x = 84.286$

$$x = \frac{84.286}{3.730}$$

$x = 22.597$ by repeatedly using the cancellation principle

and $y = 94.217$ by substituting the value 22.597 for x.

INTERPRETING

To interpret this solution, the students must recognise from the model that x is the number of kilograms of ferro-manganese and that y is the number of kilograms of pig iron needed. Hence 22.597 kg of ferro-manganese, 94.217 kg of pig iron and 5000 kg of mild steel are to be melted.

The students must also examine the assumptions made during the modelling stage. If it is not reasonable that the amount of an element should remain constant during the melting process, the model must be changed to allow for losses due to oxidation.

The students should estimate the answer to check on the feasibility of the answer calculated. For example, the amount of manganese required might be estimated to be 0.3% of 5000, that is 15 kg. Since ferro-manganese is only 70% manganese, about 20 kg of this alloy is needed. The ferro-manganese provides about 2 kg of the carbon required. Hence, the pig iron must provide the remaining 3 kg. This means that the amount of pig iron required is about 75 kg since it contains 4% carbon. In this fashion, the answers which were calculated are seen to be reasonable.

ANALYSIS OF THE MATHEMATICS

The facts, skills, strategies and concepts which the students need are listed next. An example follows.

Facts

The facts which the students are to learn include:

- the definition of percentage;
- conventions of the Cartesian co-ordinate system;
- decimal place values.

Skills

The skills which the students are to develop include:

- converting percentages to fractions;
- drawing graphs of relations;
- algebraic operations used in solving;
- calculating by hand or with a calculator.

Strategies

The strategies which the students are to learn include:

- solving two equations by elimination of a variable;
- using the assumption that the amounts of carbon and manganese remain constant during melting, as the basis of the equations which model the situation.

Concepts

The concepts which are involved include:

- knowledge about modelling a situation as a set of equations;
- knowledge about interpreting the solution;
- knowledge about solving equations.

LESSON PLANNING

The students might be motivated by emphasising the effort-saving aspects of the calculation approach, and by showing them a completely worked example. Their prior knowledge of the material, which will be needed for them to understand the new topic, should be evaluated. This may be done by discussion or by a short test containing items such as those illustrated in Figure 20.

Question 1. Write as fractions: 15% =

x% =

Question 2. Complete $0.002 \times 5000 + 200 =$ ______

Question 3. Divide 0.35 by 1000 = ______

Question 4. Write $\frac{0.7}{100}$ as a decimal fraction. ______

Question 5. Remove brackets from $2(x + 3)$ ______

Question 6. Solve for x and y:

$2x + y = 5$ $x =$ ______

$2x - y = 3$ $y =$ ______

Question 7. Solve for x and y:

$3x + 3y = 6$
$x + y = 2$
explaining your answers with a graph

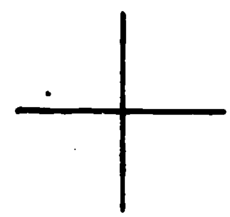


Figure 20. Test

If these items indicate that the prerequisite knowledge is lacking, it should be taught to the students before proceeding with new topics. The new material would then be presented if the students were not already familiar with the new topic.

Facts and skills

Firstly, the facts and skills would be presented using some examples. Students should become competent at skills such as:

- converting 0.53% to 0.0053;
- simplifying $3 + 2x = 4(x - 2)$ to $2x = 11$;
- transposing linear equations such as $2x - 3 = 4$;
- writing equations in the same form;
- eliminating variables between pairs of equations of the same form;
- calculating, by hand or with a calculator.

The REVIEW, INSTRUCT, RECORD, APPLY method is used.

Secondly, the <u>concepts</u> are presented.

Concrete practical exercises such as plotting graphs and substituting values in equations may be used to illustrate the concept of a simultaneous solution.

The WORKSHOP EXPERIENCE ⇄ DISCUSSION & ANALYSIS method is used.

For example, students might be required to draw the graph of the relation

$$y = x + 1$$

and observe that the points (1,2), (2,3), (3,4) and other points are solutions to the equation.

Then, they could draw the graph of the relation:

$$y = 5 - x$$

and observe that the points (3,2), (2,3), (1,4) and other points are solutions to the equation.

From the two graphs, they would be able to observe that:

- point (2,3) lies on both lines;
- $x = 2$ and $y = 3$ is the only pair of values which satisfies both equations.

The concept of simultaneous solutions might then, if time permits, be discussed in geometrical terms at the same time as the solution in Figure 21 is provided for the students.

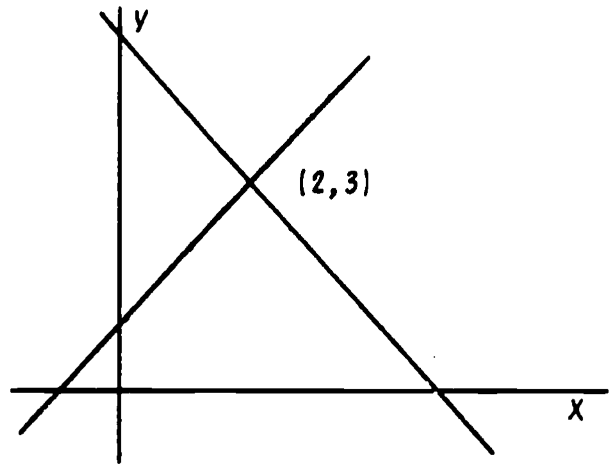


Figure 21. Geometric solution

Students must then GENERALISE and RECORD their concept graphically. They would be provided with a summary such as the one illustrated in Figure 22.

One Solution | No Solution | Many Solutions

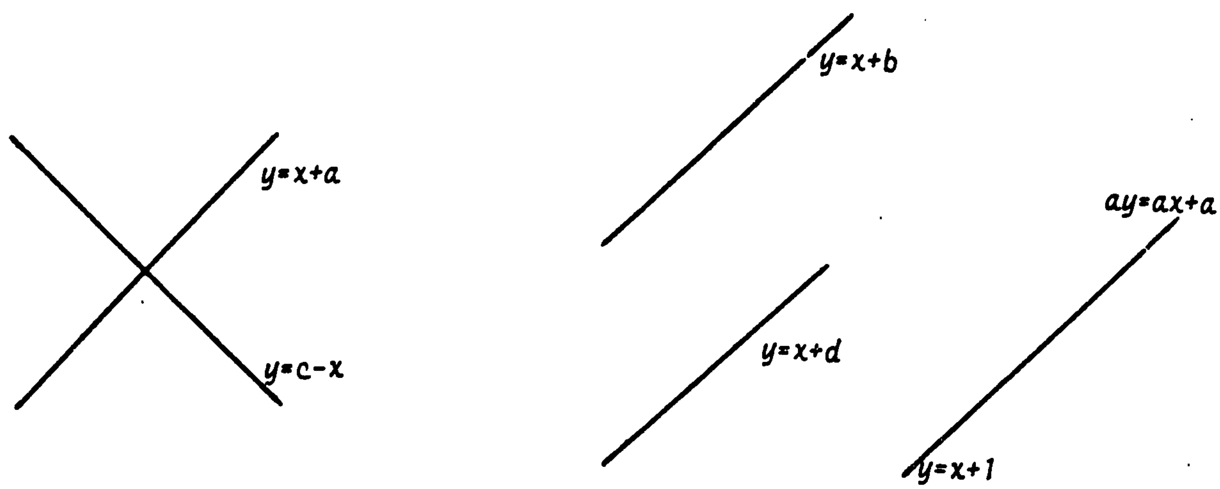


Figure 22. Summary

The students would then APPLY their knowledge to a complete example involving the solving of simultaneous equations.

Strategies

Following this, one or more of the strategies for solving pairs of equations would be presented. Students might be shown the strategy of eliminating a variable by adding, the strategy of eliminating a variable by substitution, or a graphical strategy for solving equations.

If one of these strategies has been learned by the students at school, a change in strategy at this stage would be unnecessary and possibly confusing. Hence, students could be given several pairs of equations to solve using a method of their own choosing. More able students should learn and compare all of these strategies if time permits.

Finally students might be tested using a set of practical problems which would require them to solve simple linear equations of the type just discussed.

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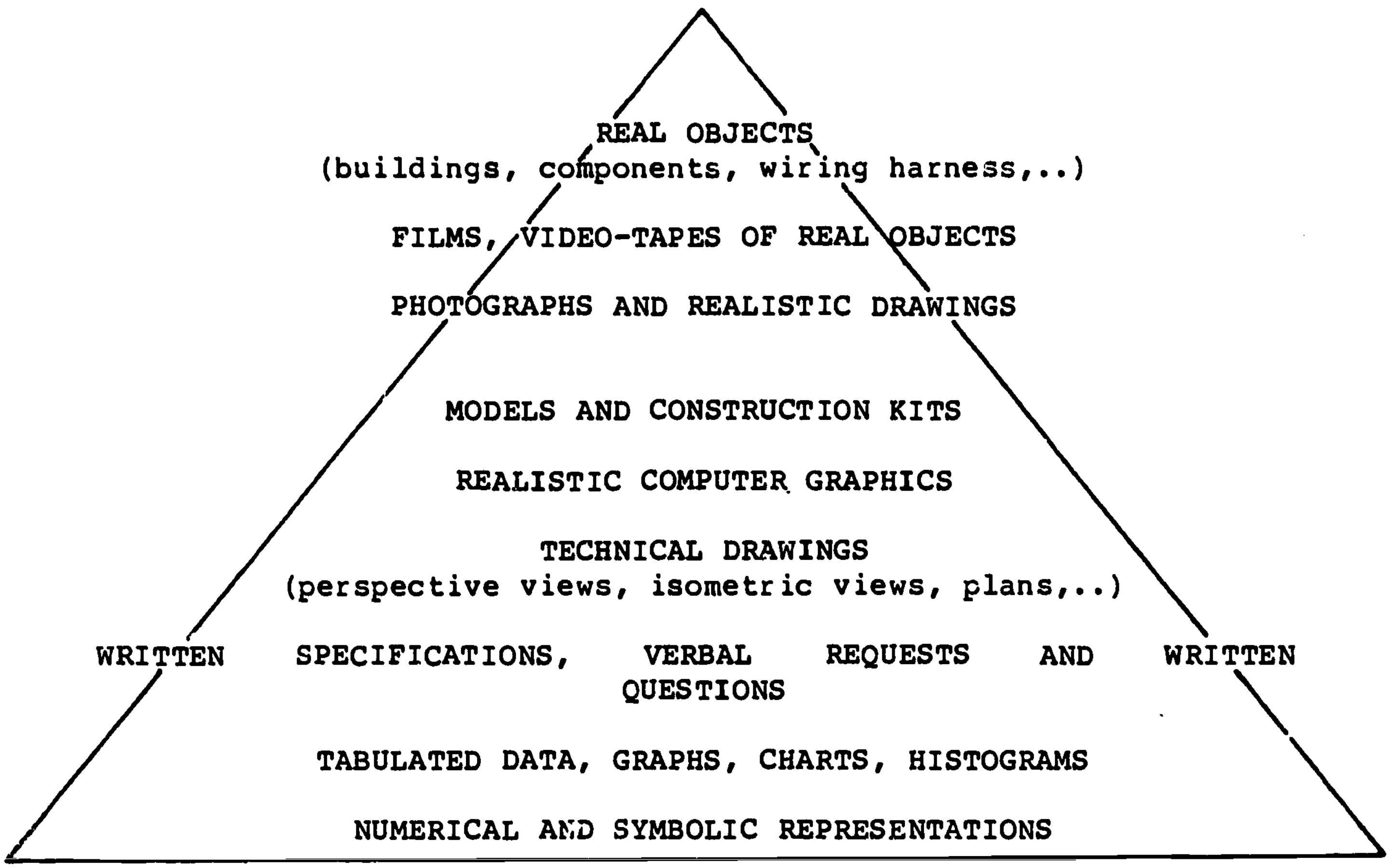
APPENDIX: THE RESOURCE WEDGE



The Resource Wedge depicted above, attempts to group resource types in a hierarchy based on their degree of concreteness or abstraction. The resource types listed lower in the wedge are seen to be more abstract than those near the top. It is suggested, therefore, that when new facts, skills, strategies or concepts are being introduced, resources should be choseh from the upper levels in the interests of the students' understanding.

NOTES:

1. The lower down the wedge, the more abstract are the ideas as a general rule.

2. The lower down the wedge, the greater is the variety of resources which are needed to achieve good results.

3. The nearer the apex or top of the wedge, the greater is the potential for language development and concept formation.

4. The learning activities and experiences nearer the top of the wedge are far less likely to:

a) disadvantage students with restricted ranges of prior experience,

b) cause confusion through inappropriate terminology and poor language skills,

c) cause mismatching of ideas with real objects in the trade context,

d) cause misconceptions, incorrect ideas or misused terminology,

e) suffer from ambiguity or misinterpretation by the TAFE lecturer and student,

f) cause perceptual difficulties and learning disorders.

5. Learning based on experiences near the top of the wedge is much more likely to:

a) promote effective thinking,

b) promote appropriate and correct terminology and language,

c) promote imagination, insight and initiative,

d) promote 'problem-solving' skills in both the developmental and applications stages,

e) provide useful evaluation situations which will enable the lecturer to develop more insights into the levels of knowledge, comprehension and application attained by individual students.